

# Surface Plasmons in Ordered and Disordered Chains of Metal Nanoparticles

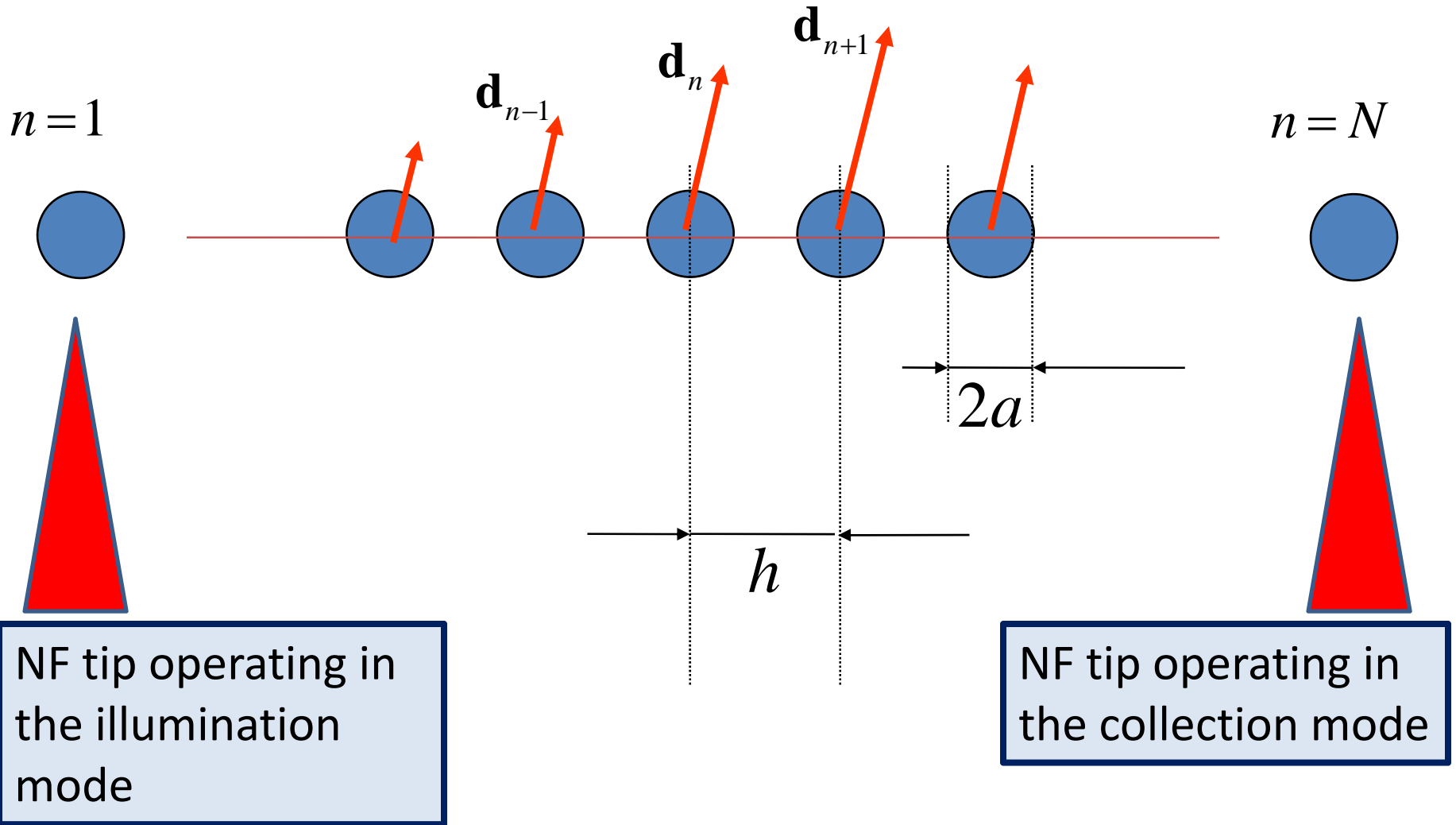
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# Physical Model



NF tip operating in the illumination mode

NF tip operating in the collection mode

# The Dipole Approximation

$$\mathbf{d}_n = \alpha_n \left[ \mathbf{E}_n + \sum_{m \neq n} \hat{G}_k(x_n, x_m) \mathbf{d}_m \right]$$


The coupled-dipole equation in the frequency domain

$$k = \frac{\omega}{c} = \text{const}$$

$\alpha_n$  - Polarizability of the  $n$ -th particle

$\hat{G}_k(x_n, x_m)$  - Green's function for the electric field in vacuum

$\mathbf{E}_n \propto \delta_{n1}$  - Electric field produced by the first tip  
(the incident field)

# Model for the Polarizability, $\alpha$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \quad [\text{Drude model for the dielectric function}]$$

$$\text{Re}\left(\frac{1}{\alpha}\right) = \frac{1}{a^3} \left[ 1 - \left(\frac{\omega}{\omega_F}\right)^2 \right]; \quad \text{Im}\left(\frac{1}{\alpha}\right) = -i \left( \frac{2k^3}{3} + \frac{1}{a^3} \frac{\omega\gamma}{\omega_F^2} \right)$$

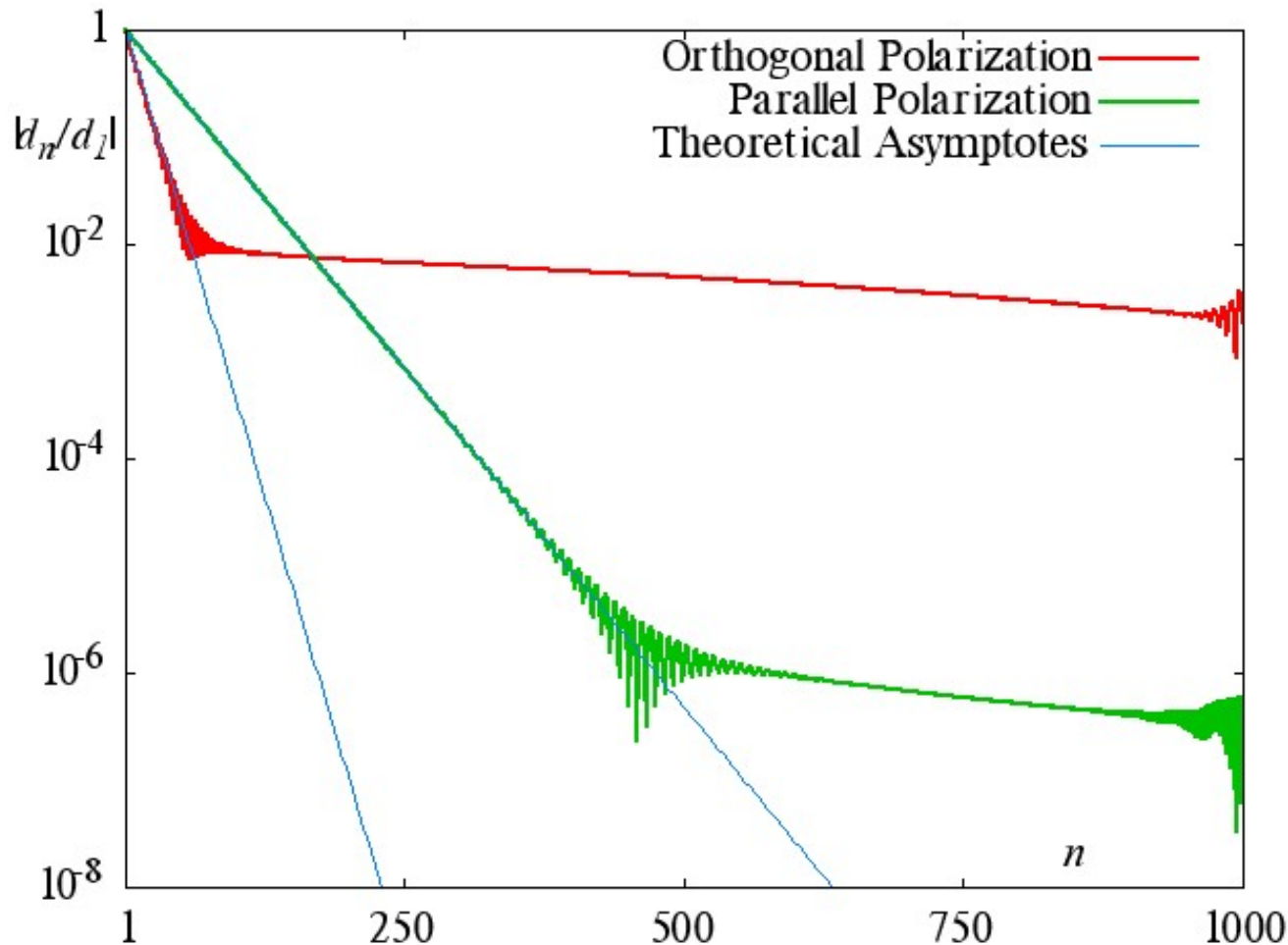
[ Quasistatic approximation + the first radiative correction ]

$a$  - Nanosphere radius

$\omega_F = \omega_p / \sqrt{3}$  - Frohlich frequency

$k = \omega / c$

# Simulation in a Finite Chain of $N=1000$ Identical Nanospheres



Parameters:

$$\omega = \omega_F$$

$$\frac{\gamma}{\omega_F} = 0.002$$

$$\lambda = \frac{2\pi c}{\omega} = 10h$$

$$h = 4a$$

# Analytical Solution in an Infinite Periodic Chain

$$d_n^{(\parallel, \perp)} = \int_{-\pi/h}^{\pi/h} \frac{E_n^{(\parallel, \perp)} e^{iqx_n}}{1/\alpha - S^{(\parallel, \perp)}(k, q)} \frac{dq}{2\pi}$$

$$S^{(\parallel, \perp)}(k, q) = 2 \sum_{n>0} G_k^{(\parallel, \perp)}(0, x_n) \cos(qx_n) \quad [\text{The dipole sum}]$$

$$\text{If } q > k, \text{ Im}[S^{(\parallel, \perp)}(k, q)] = -\frac{2k^3}{3}$$

The dispersion equation:

$$Z(k, q) = 1/\alpha - S^{(\parallel, \perp)}(k, q) = 0 \quad \Rightarrow \quad \omega = f(q)$$

# The Quasi-Particle Pole Approximation

Let, for a given value of  $k$ ,  $Z(k, q_0) \approx 0$  and  $q_0 > k$

$$Z(k, q_0) = 1/\alpha - S(k, q_0) \approx 0$$

Then we write approximately:

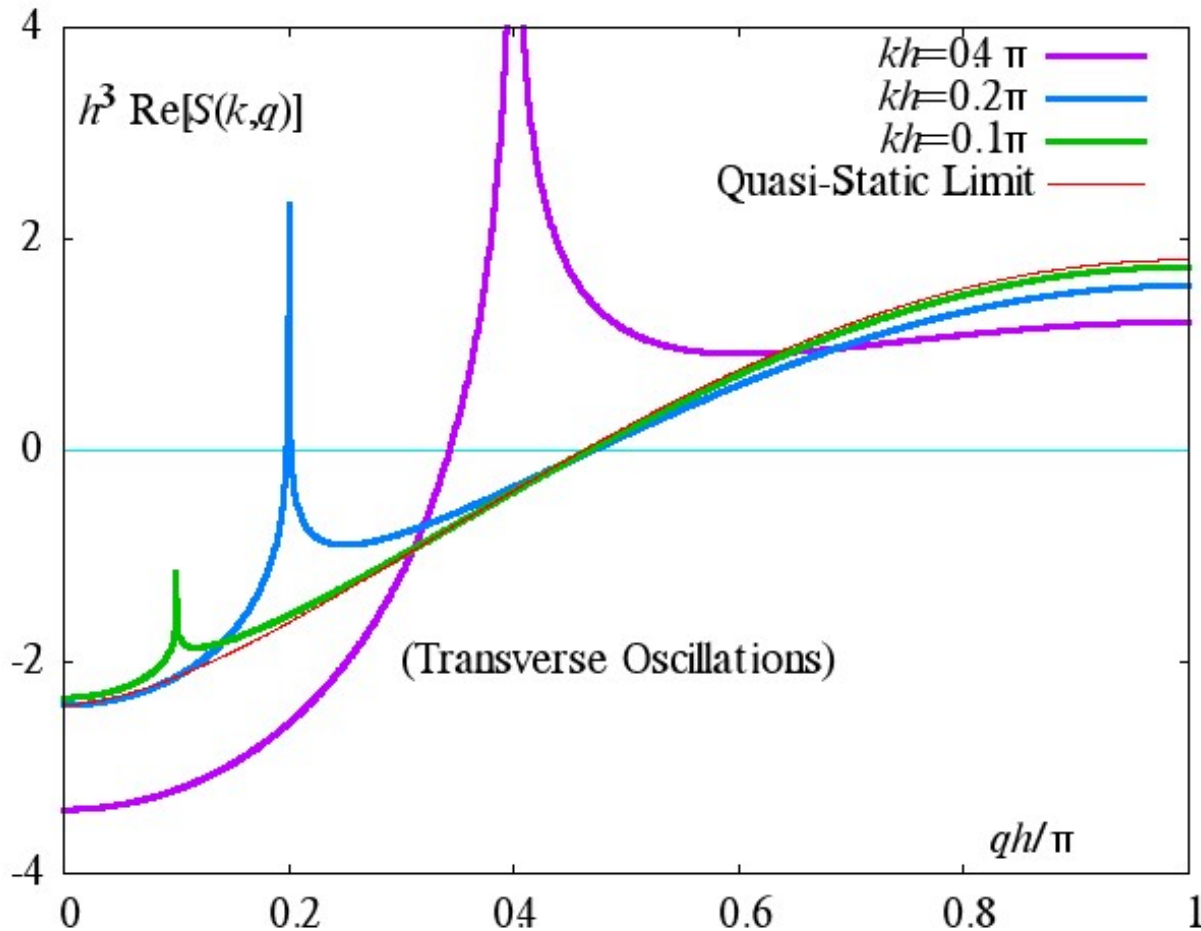
$$S(k, q) \approx \operatorname{Re} S(k, q_0) + (q - q_0) \left. \frac{\partial \operatorname{Re} S(k, q)}{\partial q} \right|_{q=q_0} - i \frac{2k^3}{3}$$



$$\ell = \frac{1}{\delta} \left. \frac{\partial \operatorname{Re} S(k, q)}{\partial q} \right|_{q=q_0} \quad [\text{Decay length}]$$

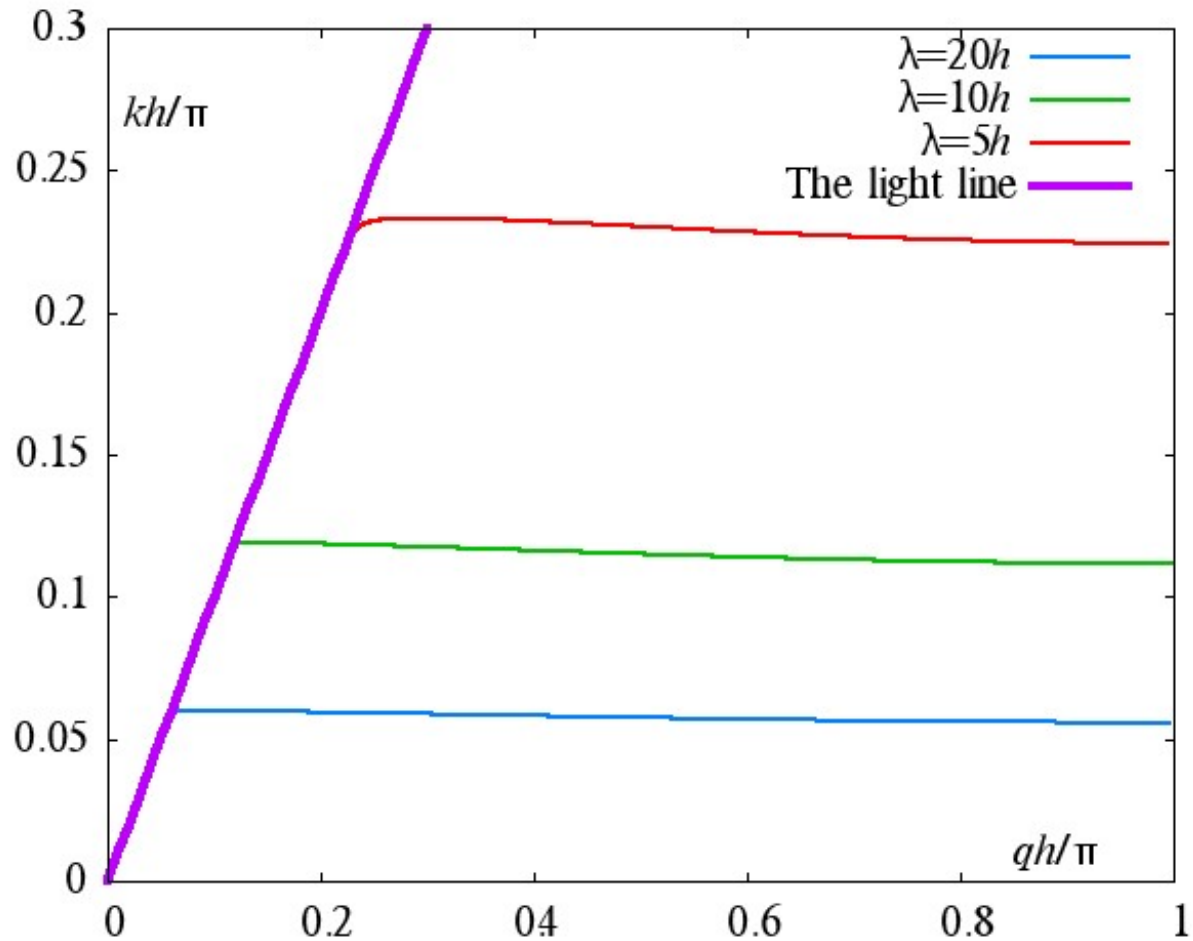
$$\delta = -\operatorname{Im}(1/\alpha) - 2k^3/3 \quad [\text{Absorption strength}]$$

# The Dipole Sum (Transverse Oscillations in an Infinite Periodic Chain)

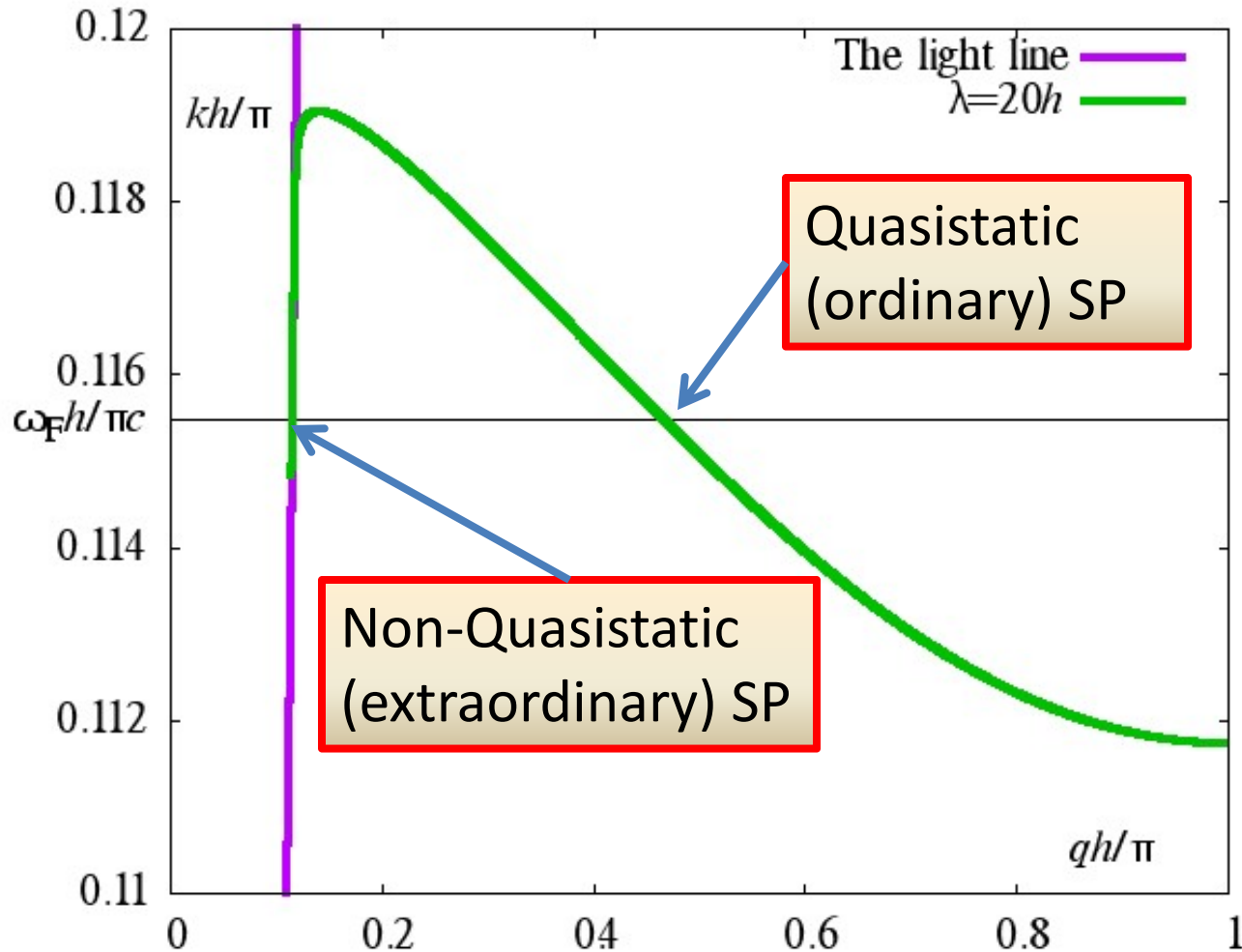




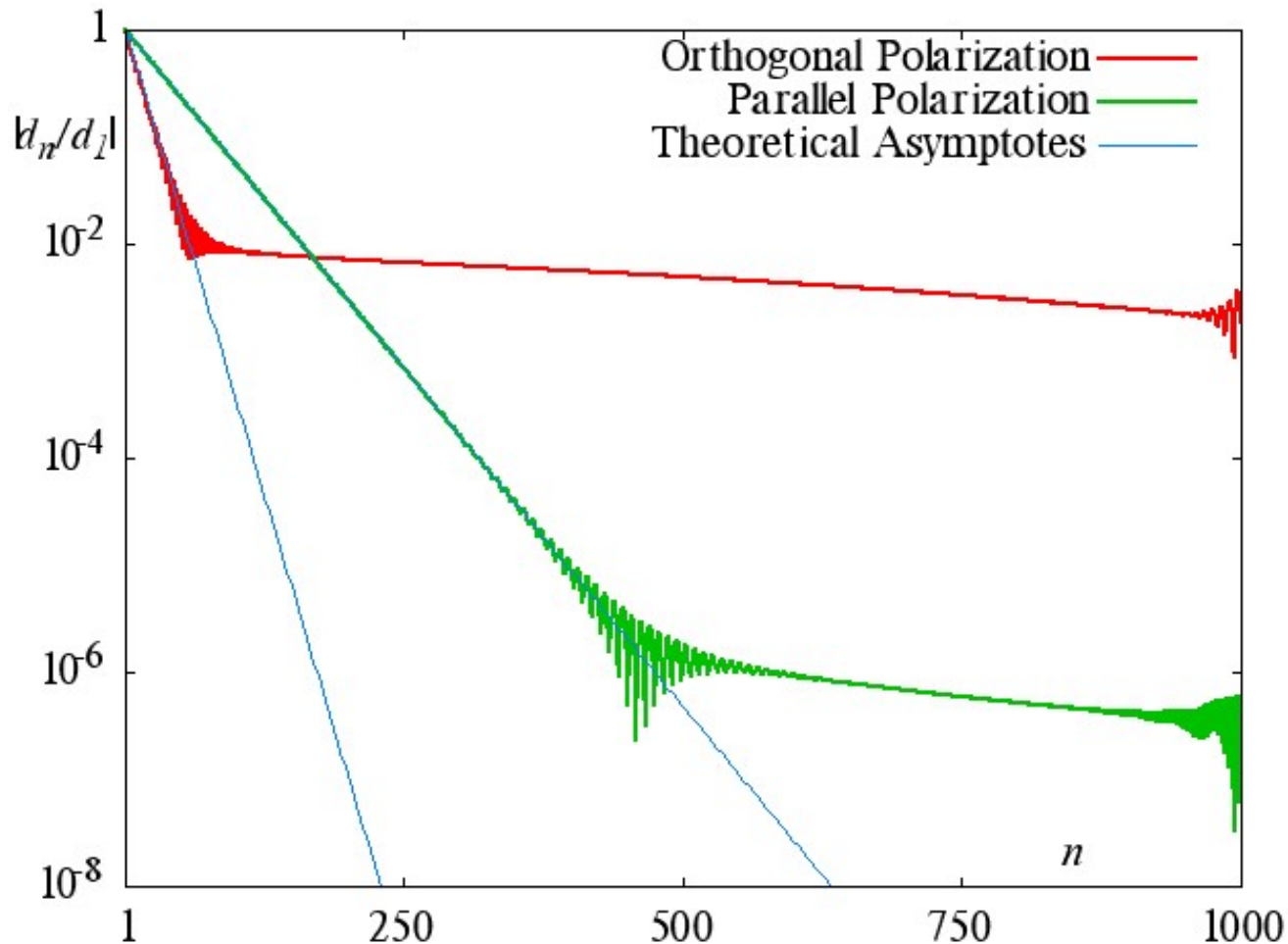
# The Dispersion Curve (Transverse Oscillations in an Infinite Ordered Chain)



# Dispersion Curves (continued)



# Simulation in a Finite Chain of $N=1000$ Identical Nanospheres



Parameters:

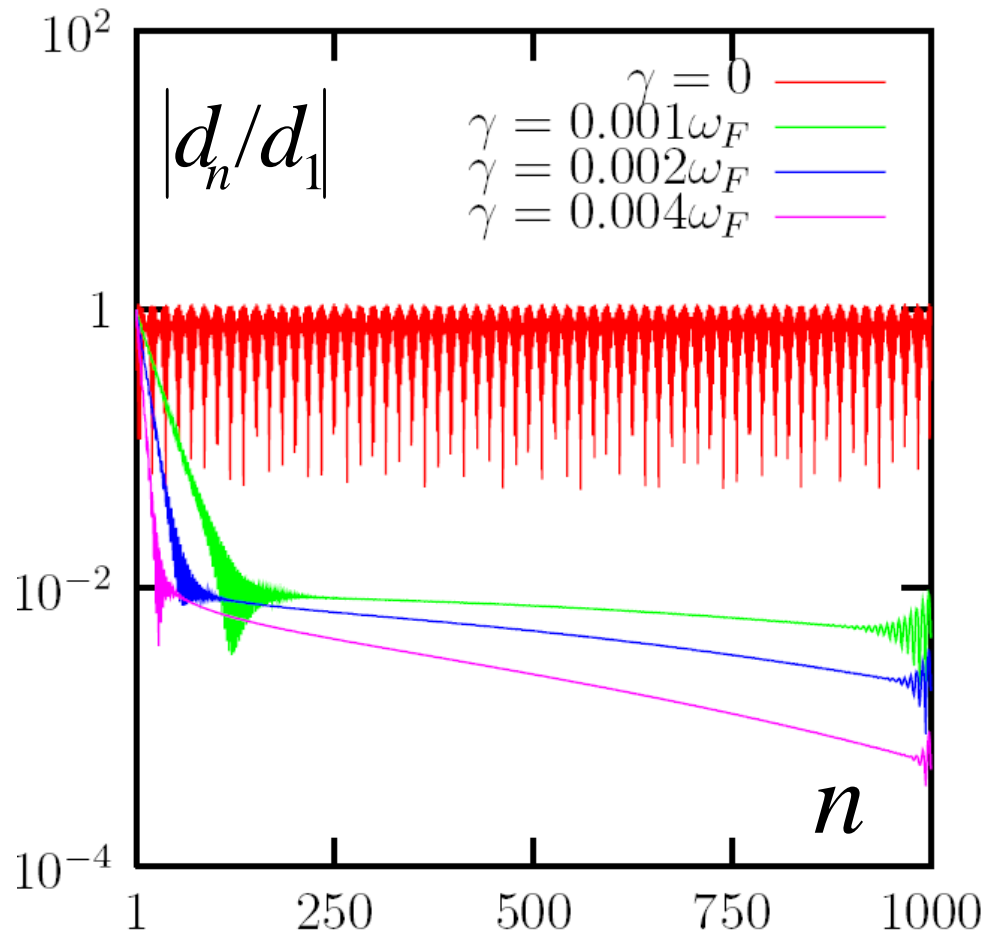
$$\omega = \omega_F$$

$$\frac{\gamma}{\omega_F} = 0.002$$

$$\lambda = \frac{2\pi c}{\omega} = 10h$$

$$h = 4a$$

# Effect of Ohmic Losses



Parameters:

$$\omega = \omega_F$$

$\frac{\gamma}{\omega_F}$  varies

$$\lambda = \frac{2\pi c}{\omega} = 10h$$

$$h = 4a$$

# Effects of Disorder

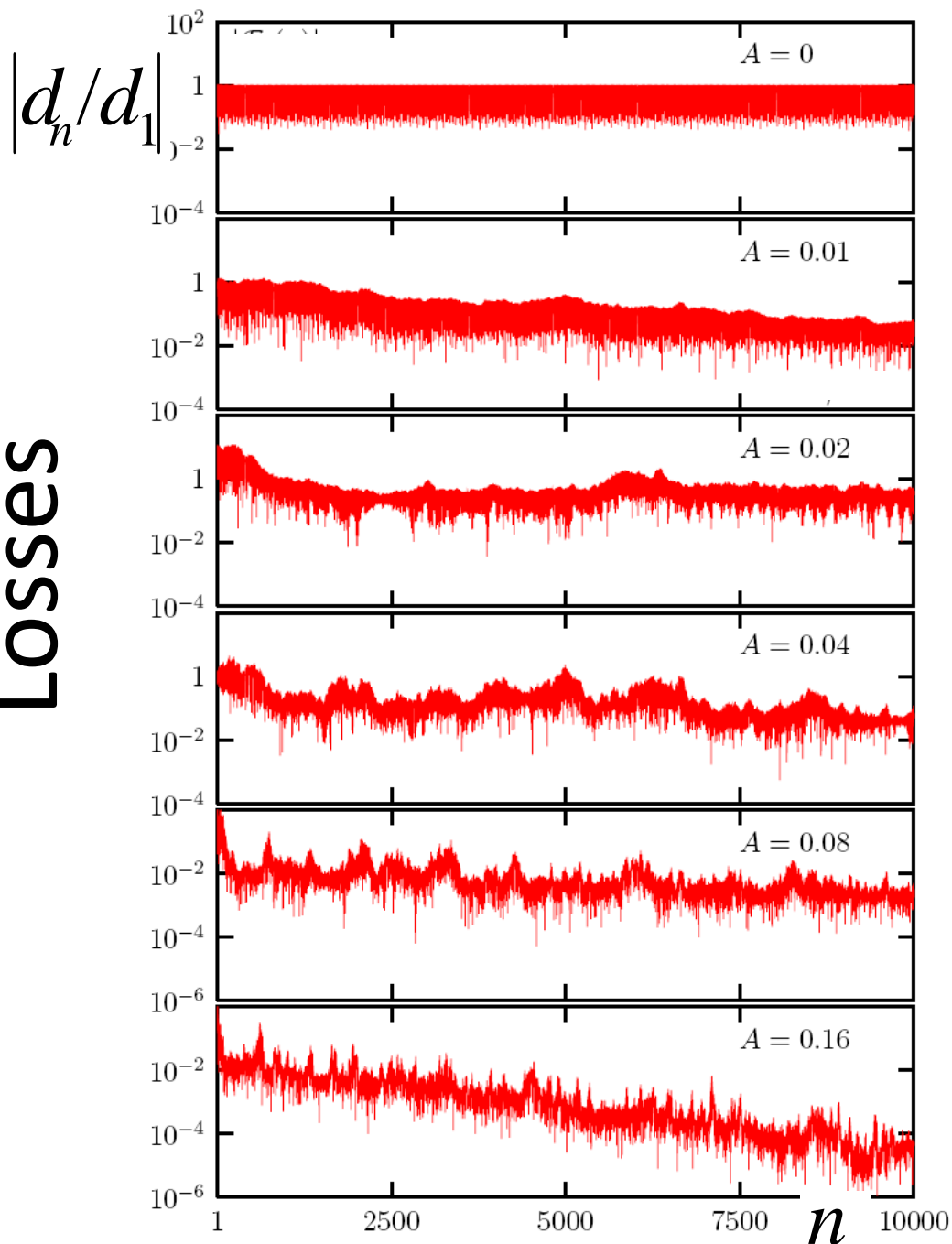
- Off-diagonal disorder (disorder in the nanoparticle positions)

We assume here that the position of the  $n$ -th particle is evenly distributed in the interval  $[h(n-A), h(n+A)]$ ,  $A \ll 1$

- Diagonal disorder  
[A more subtle effect, not considered in this talk; see Phys.Rev.B **75**, 085426 (2007)]

# Off-Diagonal Disorder in the Absence of Ohmic Losses

Losses



Parameters:

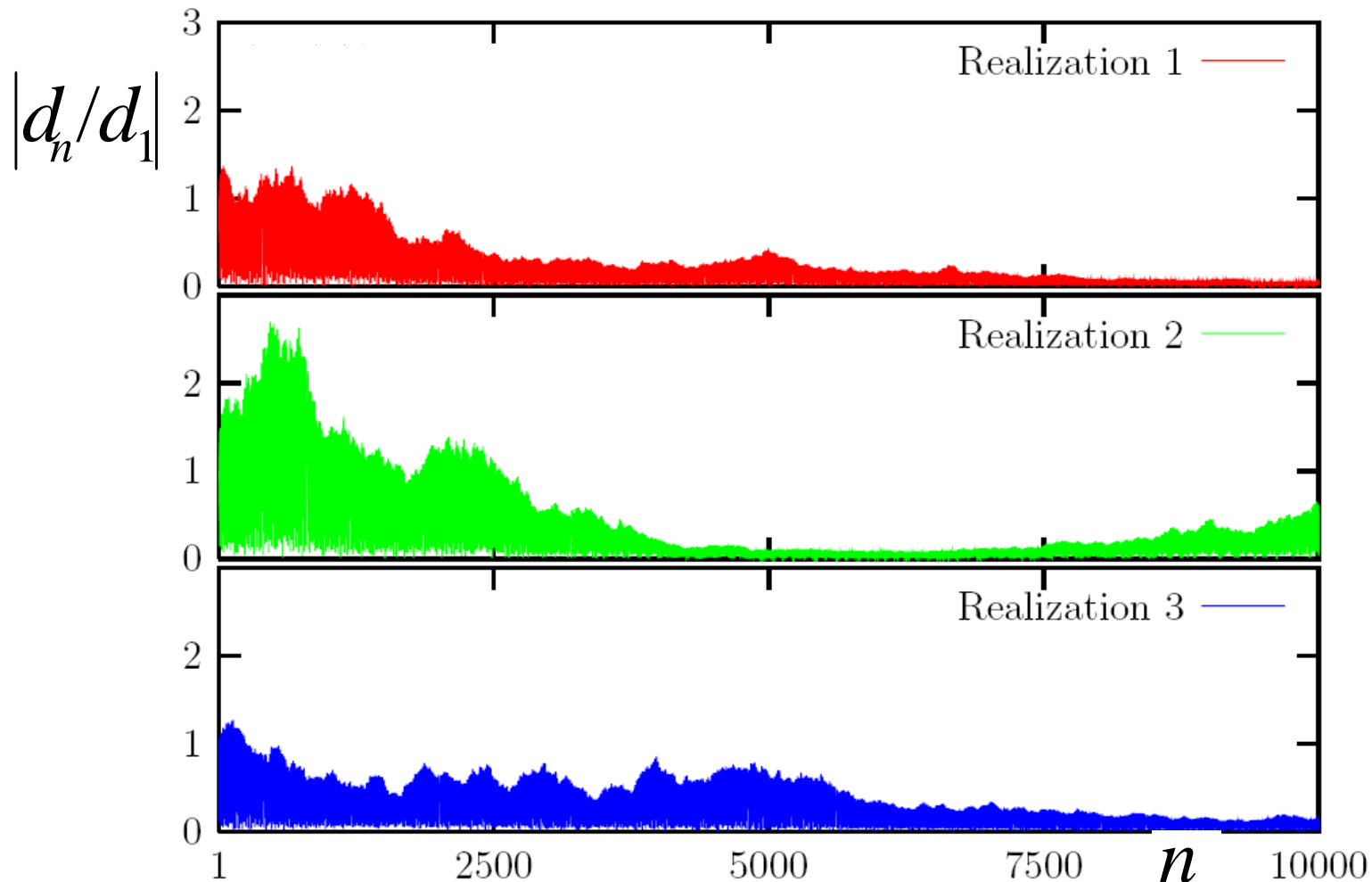
$$\omega = \omega_F$$

$$\frac{\gamma}{\omega_F} = 0$$

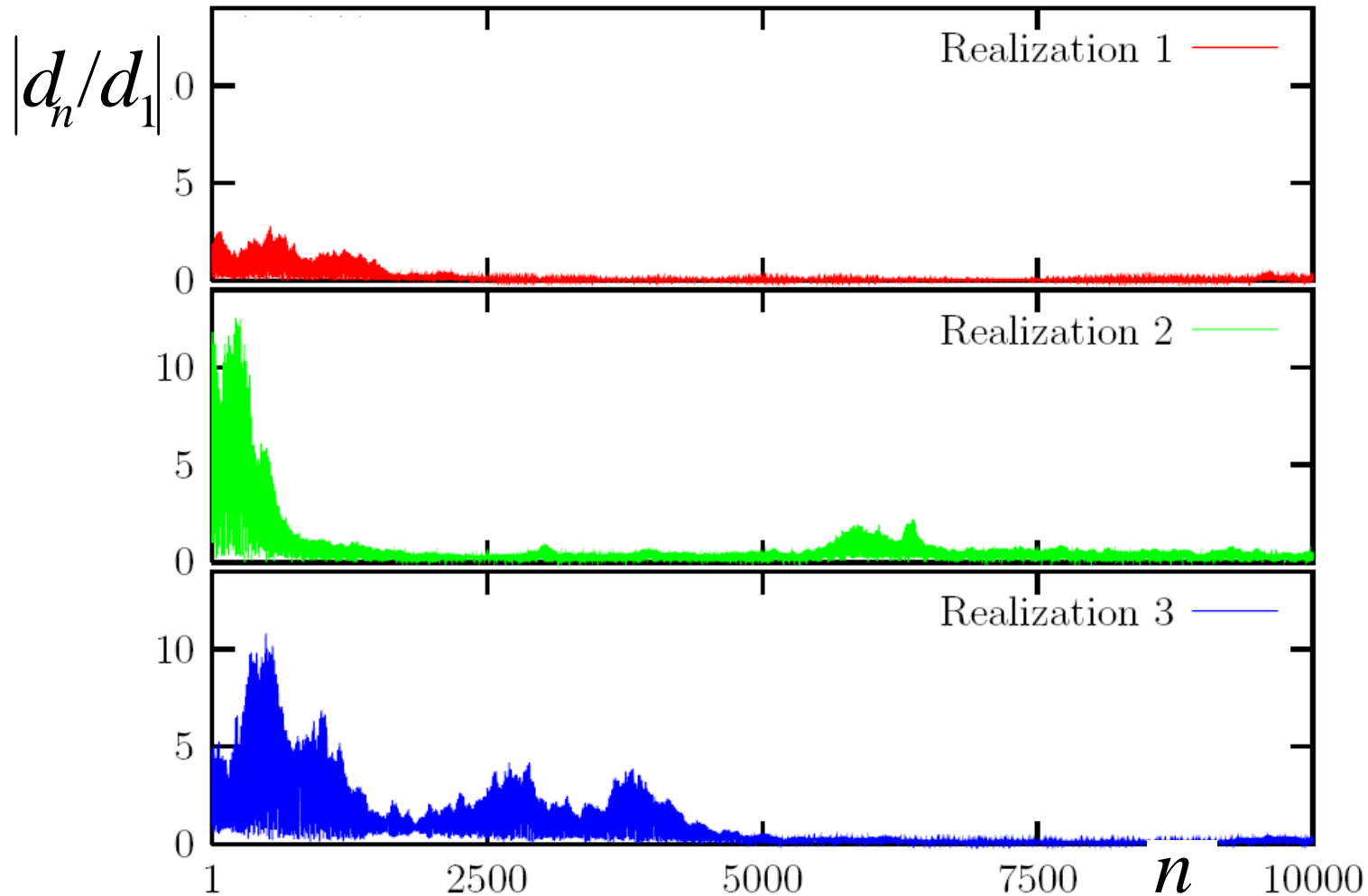
$$\lambda = \frac{2\pi c}{\omega} = 10h$$

$$h = 4a$$

# Different Realization of Disorder at the Level $A=0.01$



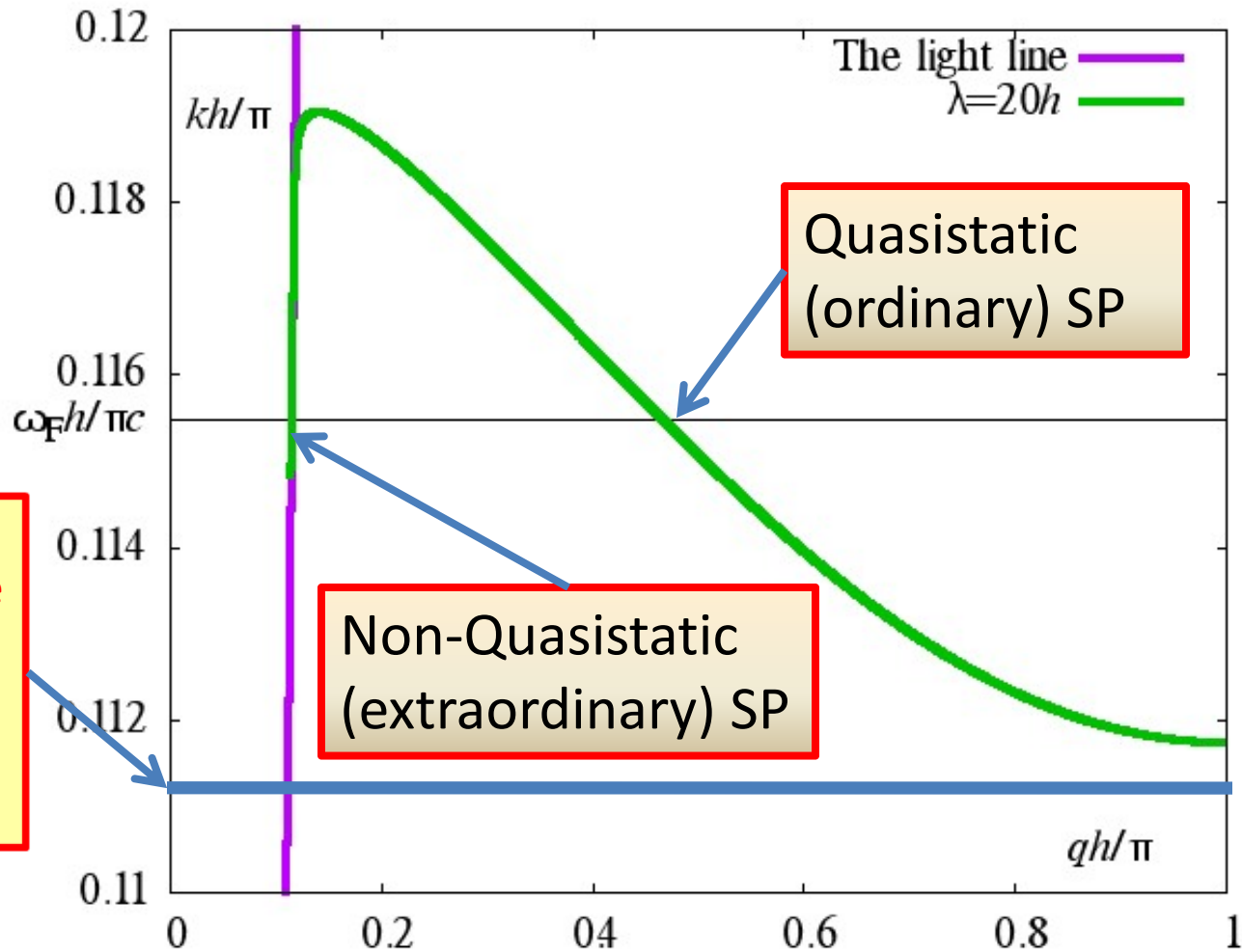
# Different Realization of Disorder at the Level $A=0.02$



$$\omega = \omega_F$$



# Non-Quasistatic SP at Different Levels of Disorder

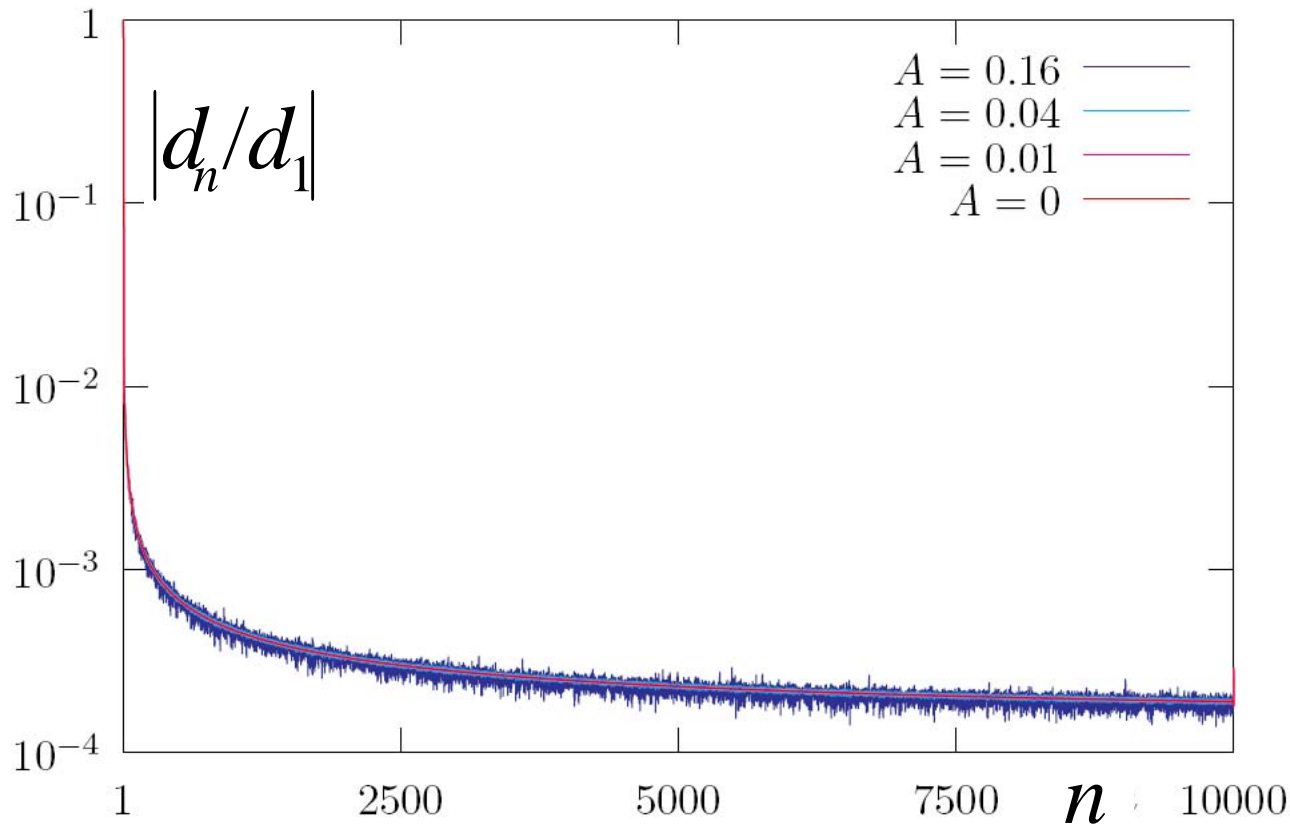


Let the frequency be small enough so that the ordinary SP is not excited

Quasistatic (ordinary) SP

Non-Quasistatic (extraordinary) SP

# Non-Quasistatic SP at Different Levels of Disorder (continued)

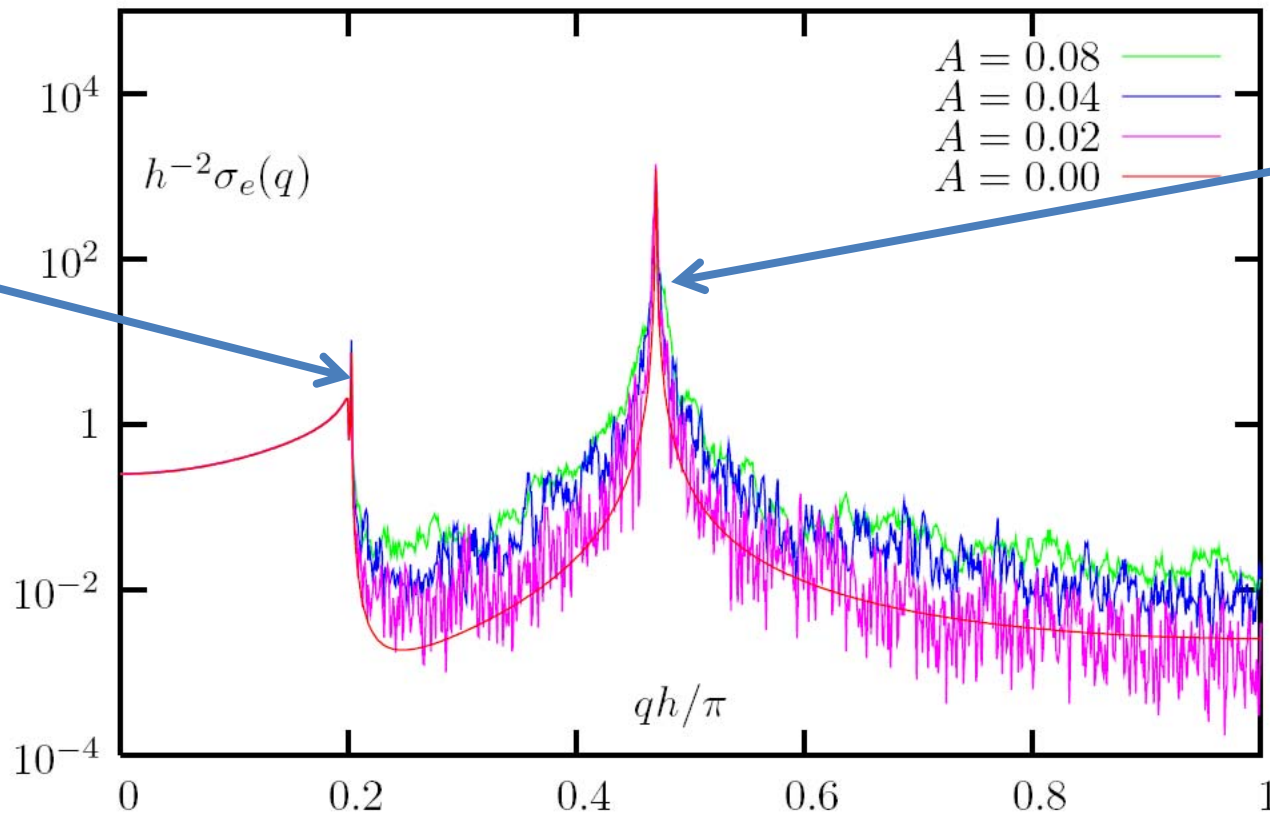


$$\frac{\omega}{\omega_F} = 0.984$$

(small enough  
so that the  
ordinary SP  
is not excited)

# Specific Extinction for Excitation by a Plane Wave $\exp(iqx)$

(e.g., created by the TIR,  $q$  can be larger than  $k$ )



Extraordinary  
(non-quasistatic)  
SP

Ordinary  
(quasistatic) SP

# CONCLUSIONS

- Extraordinary SP can propagate to remarkable distances and is affected by the off-diagonal disorder only weakly
- This is in contrast to the ordinary SP which is dramatically affected by Ohmic losses and disorder
- Numerical evidence indicates that the ordinary SP becomes strongly localized in the presence of disorder while the extraordinary SP does not.