Optical tomography with structured illumination

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We consider the image reconstruction problem for optical tomography with structured illumination. A fast image reconstruction algorithm is proposed that reduces the required number of measurements of the optical field compared to methods that utilize point-source illumination. The results are illustrated with numerical simulations. © 2009 Optical Society of America

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Optical tomography is an emerging biomedical imaging modality that uses diffuse light to probe structural variations in the optical properties of tissue [1]. In a typical experiment, a highly scattering medium is illuminated by a point source, and the resultant transmitted light is collected by an optical fiber or imaged onto a CCD array. In the latter case, if a narrow collimated beam is used as the source of illumination, very large data sets of 10^8-10^{10} source–detector pairs may be acquired. The method is then known as noncontact optical tomography [2–4], and the corresponding inverse scattering problem has been the subject of considerable investigation [5,6]. In particular, fast image reconstruction algorithms have been developed for several geometries and experiments have been performed that demonstrate the ability to image complex structures with subcentimeter resolution [7]. The reconstruction algorithms are formulated in the spatial-frequency domain and utilize a relatively small number of Fourier components of the scattering data. However, a large number of point sources are necessary to synthesize the required frequency components, resulting in prolonged data collection times.

In a recent work, Cuccia *et al.* [8] introduced the powerful experimental technique of modulated imaging, which uses structured illumination for *direct visualization* of absorbing inhomogeneities in a turbid medium. Since high spatial frequencies decay exponentially with propagation, the method allows for a simple means to achieve optical sectioning by adjusting the spatial frequency of the illuminating field. The intensity images that are obtained in this manner contain information about the medium. However, they are not tomographic, nor are they quantitatively related to the medium's optical properties.

In this Letter, we propose an alternative to noncontact optical tomography in which, by making use of structured illumination, scattering data are *directly measured* in the spatial-frequency domain. We then formulate the relevant inverse scattering problem and show that it is possible to devise a fast reconstruction algorithm to recover the optical absorption coefficient for both reflection and transmission experiments in the slab geometry. The principal advantage of the proposed method—a hybrid of modulated imaging and a fast image reconstruction algorithm—is that it allows for rapid collection of experimental data compared to the usual point-source approach to noncontact optical tomography.

We begin by considering the propagation of a diffuse wave in an absorbing medium. The energy density of the wave is assumed to satisfy the time-independent diffusion equation [6]

$$-D\nabla^2 u(\mathbf{r}) + \alpha(\mathbf{r})u(\mathbf{r}) = S(\mathbf{r}), \tag{1}$$

where α is the absorption coefficient, D is the diffusion constant, and S is the power density of a continuous-wave source. The energy density is also taken to obey the boundary condition

$$u(\mathbf{r}) + \ell \hat{\mathbf{n}} \cdot \nabla u(\mathbf{r}) = 0 \tag{2}$$

on the surface bounding the medium, where $\hat{\mathbf{n}}$ is the outward unit normal and ℓ is the extrapolation length. The intensity measured by a point detector at $\mathbf{r}$ is given by the expression

$$I(\mathbf{r}) = \frac{c}{4\pi}\left(1 + \frac{\ell^*}{\ell}\right)\int G(\mathbf{r},\mathbf{r}')S(\mathbf{r}')d^3r', \tag{3}$$

where G is the Green's function for Eq. (1), we have assumed that the detector collects light in the outward normal direction and ℓ^* is the transport mean free path, which is related to the diffusion constant by $D=1/3c\ell^*$. Within the accuracy of the first Born approximation, the Green's function is given by

$$G(\mathbf{r},\mathbf{r}') = G_0(\mathbf{r},\mathbf{r}') - \int G_0(\mathbf{r},\mathbf{r}'')\delta\alpha(\mathbf{r}'')G_0(\mathbf{r}'',\mathbf{r}')d^3r'', \tag{4}$$

where G_0 is the Green's function for Eq. (1) with $\alpha(\mathbf{r})=\alpha_0$ and $\delta\alpha(\mathbf{r})=\alpha(\mathbf{r})-\alpha_0$.

For the remainder of this Letter, we assume that the medium to be imaged is a slab of width L, a geometry that is often employed in optical mammography. We also assume that the source of illumination is confined to the plane $z=0$ and that transmitted light is detected on the plane $z=L$. We will also be interested in measurements for which reflected light is detected on the $z=0$ plane. In either case, we consider a source whose power density is of the form

$$S(\mathbf{r}) = S_0(1 + A\cos(\mathbf{Q}\cdot\boldsymbol{\rho} + \phi))\delta(z), \tag{5}$$

where $\mathbf{r}=(\boldsymbol{\rho},z)$; $\mathbf{Q}$ is a two-dimensional wave vector; $\delta(z)$ is the one-dimensional Dirac delta function; and S_0, A, and ϕ are the amplitude, modulation depth, and phase of the source, respectively. Such a source, which can be constructed by employing a liquid-crystal spatial light modulator, produces a type of structured illumination in which the incident diffuse wave is modulated with spatial frequency $\mathbf{Q}$. Next, we consider two separate measurements with phases $\phi=0$, $\pi/2$ and high-pass filter the results, eliminating the zero-frequency contribution to the field. We then find that by taking an appropriate linear combination of the corresponding intensities, the change in intensity of a diffuse wave due to fluctuations in the absorption of the medium is proportional to the data function φ, which is defined by

$$\varphi(\mathbf{r}) = \int d^3r' d^2\rho'' e^{i\mathbf{Q}\cdot\boldsymbol{\rho}''} G_0(\mathbf{r},\mathbf{r}')G_0(\mathbf{r}';\boldsymbol{\rho}'',0)\,\delta\alpha(\mathbf{r}'), \tag{6}$$

where $\mathbf{r}$ is the point of observation and we have retained only the spatially oscillating part of the intensity.

The inverse problem consists of recovering the function $\delta\alpha$ from boundary measurements of φ as the source wave vector $\mathbf{Q}$ is varied. We assume that φ is sampled on a square lattice with spacing h. To proceed, we introduce the lattice Fourier transform of φ, which is defined by

$$\tilde{\varphi}(\mathbf{q},z) = \sum_{\boldsymbol{\rho}} \exp(i\mathbf{q}\cdot\boldsymbol{\rho})\varphi(\boldsymbol{\rho},z), \tag{7}$$

where $\mathbf{q}$ belongs to the first Brillouin zone (FBZ) of the lattice [9]. Next, we make use of the plane-wave decomposition of the Green's function, which is of the form

$$G_0(\mathbf{r},\mathbf{r}') = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')} g(z,z';q). \tag{8}$$

Here g is given by the expression [6]

$$g(z,z';q) = \frac{\ell}{D}\frac{\sinh[Q(q)(L-|z-z'|)] + Q(q)\ell\cosh[Q(q)(L-|z-z'|)]}{\sinh(Q(q)L) + 2Q(q)\ell\cosh(Q(q)L) + (Q(q)\ell)^2\sinh(Q(q)L)}, \tag{9}$$

where $Q(q)=\sqrt{q^2+\alpha_0/D}$. We then find that

$$\tilde{\varphi}(\mathbf{q},\mathbf{Q}) = \frac{1}{h^2}\int_0^L g(0,z;Q)g(z,z_d;q)\widetilde{\delta\alpha}(\mathbf{q}+\mathbf{Q},z)dz, \tag{10}$$

where z_d is the z coordinate of the detector, we have assumed that $\delta\alpha$ is bandlimited to the FBZ and have indicated the dependence of φ on the source wave vector $\mathbf{Q}$ explicitly. After a change of variables, we rewrite Eq. (10) in the form

$$\psi(\mathbf{Q},\mathbf{q}) = \int_0^L K(\mathbf{Q},z;\mathbf{q})\widetilde{\delta\alpha}(\mathbf{q},z)dz, \tag{11}$$

where

$$K(\mathbf{Q},z;\mathbf{q}) = \frac{1}{h^2}g(0,z;Q)g(z,z_d;|\mathbf{Q}-\mathbf{q}|), \tag{12}$$

$$\psi(\mathbf{Q},\mathbf{q}) = \tilde{\varphi}(\mathbf{q}-\mathbf{Q},\mathbf{Q}). \tag{13}$$

For fixed $\mathbf{q}$, Eq. (11) defines a system of one-dimensional integral equations for the Fourier transform $\widetilde{\delta\alpha}(\mathbf{q},z)$. Following the general approach of Ref. [6], we compute the pseudoinverse solution of Eq. (11) and perform an inverse Fourier transform to obtain our main result, which is the inversion formula

$$\delta\alpha(\mathbf{r}) = \int_{FBZ}\frac{d^2q}{(2\pi)^2}e^{-i\mathbf{q}\cdot\boldsymbol{\rho}}\sum_{\mathbf{Q},\mathbf{Q}'}K^*(\mathbf{Q},z;\mathbf{q})M^{-1}_{\mathbf{Q}\mathbf{Q}'}(\mathbf{q})\psi(\mathbf{Q}',\mathbf{q}). \tag{14}$$

Here the matrix elements of M are defined by

$$M_{\mathbf{Q}\mathbf{Q}'}(\mathbf{q}) = \int_0^L K(\mathbf{Q},z;\mathbf{q})K^*(\mathbf{Q}',z;\mathbf{q})dz \tag{15}$$

and the summations are carried out over all directions of illumination. It is important to note that M^{-1} must be regularized to control the ill-posedness of the inverse problem; this we do by computing the singular value decomposition of M and truncating all singular values below a fixed threshold. Finally, an image reconstruction algorithm based on Eq. (14), with the use of the fast Fourier transform to compute $\tilde{\varphi}$, has computational complexity $O(NM\log M)$, where M is the number of measurements and N is the number of source wave vectors.

To illustrate the use of the inversion formula (14), we numerically simulated the reconstruction of $\delta\alpha$ for a medium with optical properties similar to breast tissue in the near infrared. We chose the background absorption and diffusion constants to be $\alpha_0 = 1\ \text{ns}^{-1}$ and $D = 1\ \text{cm}^2\,\text{ns}^{-1}$, respectively. The slab thickness was set to be $L = 6.1$ cm and the extrapolation length to be $\ell = 0.1$ cm. The detectors were located on a square lattice with spacing $h = 0.1$ cm. The source wave vectors $\mathbf{Q}$ were chosen to occupy an 11×11 square lattice and were arranged symmetrically around the center of the FBZ with a spacing of $1.2\ \text{cm}^{-1}$. The forward data were computed for a collection of point absorbers, allowing for interactions between the absorbers, as described in Ref. [10]. The absorbers had an effective volume of $10^{-3}\ \text{cm}^3$ and a contrast of 4:1 and were arranged in two parallel planes. In the $z = 2$ cm plane, a pair of absorbers were placed at $x = y = 0.7$ cm and $x = y = -0.7$ cm, and in the $z = 4$ cm plane, a second pair of absorbers was placed at $x = -y = 0.7$ cm and $x = -y = -0.7$ cm. Thus the first pair of absorbers is rotated by $\pi/2$ with respect to the second.

Tomographic images were reconstructed with a $5.1\ \text{cm} \times 5.1\ \text{cm}$ field of view and a pixel size of 0.1 cm. Slices are shown in the planes $z = 1, 2, \ldots, 6$ cm. In Fig. 1 we present our results for the slab geometry. It can be seen that the resolution, as measured by the full width at half-maximum (FWHM), is 0.4 cm in the transverse direction and 0.9 cm in the depth direction, the results being approximately the same for both planes in which the absorbers are present. Reconstructions in the half-space geometry (with $L \to \infty$), shown in Fig. 2, are less well resolved than in the case of the slab, as is usual [11]. The FWHM in the $z = 2$ cm plane is 0.8 cm in the transverse direction and 1.0 cm in the depth direction. The absorbers in the $z = 4$ cm plane are not visible at the scale shown. If the absorbers in the $z = 2$ cm plane are removed, reconstruction at $z = 4$ cm becomes possible, albeit with less accuracy (image not shown). We note that the resolution limits we have obtained must be considered to be best-case estimates. However, comparisons of simulations with noncontact optical tomography experiments indicate that under realistic conditions, ill-posedness is a more significant determinant of image resolution than instability due to the effects of noise or systematic errors [4,7].

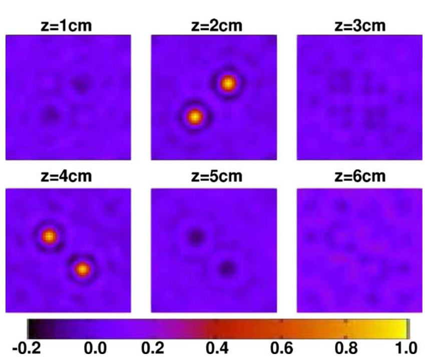

Fig. 1. (Color online) Reconstruction of $\delta\alpha$ in the slab geometry. All images are plotted on the same linear color scale.

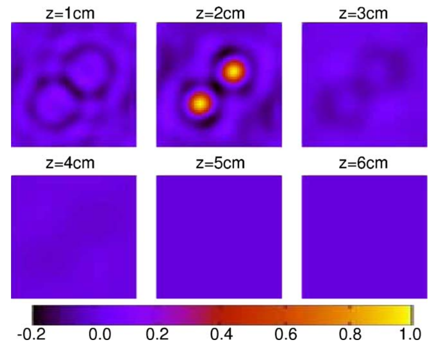

Fig. 2. (Color online) Reconstruction of $\delta\alpha$ in the half-space geometry. All images are plotted on the same linear color scale.

It is instructive to contrast the above results with those that can be obtained with point-source illumination. In Ref. [7], a noncontact optical tomography system with 1225 sources was used to reconstruct absorption images with resolution comparable to the results shown in Fig. 1. However, using structured illumination, only 2×121 incident wave vectors (accounting for both phases $\phi = 0$, $\pi/2$) were needed to obtain the equivalent Fourier components. Thus, it can be seen that comparable performance may be achieved with nearly a factor of five speed-up in data collection time, assuming equivalent integrated source power.

In conclusion, we have developed a fast image reconstruction algorithm for optical tomography with structured illumination. The proposed algorithm achieves high spatial resolution while simultaneously reducing the required number of measurements of the optical field compared to methods that utilize point-source illumination.

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References

1. S. R. Arridge, Inverse Probl. **15**, R41 (1999).
2. R. Schulz, J. Ripoll, and V. Ntziachristos, Opt. Lett. **28**, 1701 (2003).
3. G. Turner, G. Zacharakis, A. Soubret, J. Ripoll, and V. Ntziachristos, Opt. Lett. **30**, 409 (2005).
4. Z.-M. Wang, G. Y. Panasyuk, V. A. Markel, and J. C. Schotland, Opt. Lett. **30**, 3338 (2005).
5. V. A. Markel and J. C. Schotland, Appl. Phys. Lett. **81**, 1180 (2002).
6. V. A. Markel and J. C. Schotland, Phys. Rev. E **70**, 056616 (2004).
7. S. D. Konecky, G. Y. Panasyuk, K. Lee, V. Markel, A. G. Yodh, and J. C. Schotland, Opt. Express **16**, 5049 (2008).
8. D. Cuccia, F. Bevilacqua, A. Durkin, and B. Tromberg, Opt. Lett. **30**, 1354 (2005).
9. The FBZ is defined to be $[-\pi/h, \pi/h] \times [-\pi/h, \pi/h]$, where h is the spacing of the detector lattice.
10. A. R. Fisher, A. J. Schissler, and J. C. Schotland, Phys. Rev. E **76**, 036604 (2007).
11. V. A. Markel, V. Mital, and J. C. Schotland, J. Opt. Soc. Am. A **20**, 890902 (2003).