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On the sign of refraction in anisotropic non-magnetic media

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Abstract
The main message of this paper is that refraction of a narrow collimated beam at a negative angle into strongly anisotropic non-magnetic crystals—a phenomenon which was observed in a number of experiments—should not be confused with negative refraction and lacks important physical characteristics of the latter. In particular, it is shown that there is no contradiction between the theory previously developed by one of the authors (in Markel 2008 *Opt. Express* **16** 19152), where it is claimed that negative refraction is impossible, and the experiments mentioned above.

Keywords: layered media, anisotropy, negative refraction, homogenization

(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is frequently stated in the literature that negative refraction is a well-established phenomenon in non-magnetic anisotropic media [1, 2]. In particular, refraction of a narrow collimated beam at a negative angle was convincingly demonstrated experimentally in a YVO_4 bicrystal [3] and in an artificially grown semiconductor layered medium [4]. It has also been proposed theoretically that negative refraction can occur in nematic liquid crystals [5]. On the other hand, one of the authors has recently argued that, in isotropic media, negative refraction is not achievable at all while in anisotropic media it is only possible for evanescent waves [6].

In this paper, these two points of view will be reconciled. We will show that the negative beam deflection observed and discussed in [3–5] cannot be classified as negative refraction, at least not according to the classical definition which was used in [6]. It is rather a special case of the more general phenomenon of counterposition [7–10]. According to Lakhtakia and McCall, counterposition takes place when the wavevector of a refracted plane wave and the direction of energy propagation lie on the opposite sides of the interface normal. This phenomenon was also discussed by Boardman [11]. Note that counterposition can also occur due to uniform motion of the medium [9].

In this paper we will show that the phenomenon of counterposition in non-magnetic, strongly anisotropic crystals lacks certain very important physical characteristics of true negative refraction. In particular, anisotropic non-magnetic crystals are not capable of all-angle exponential amplification of the transmission coefficient for evanescent waves, except in the limited sense, as discussed in section 5 below. This amplification is an indispensable ingredient of the so-called 'superlens' as it was originally proposed by Pendry [12]. Note that by the term 'superlens' we mean here the device which is, at least theoretically, capable of exponential amplification of evanescent waves with a broad distribution of lateral wavenumbers, not the so-called 'poor man's superlens' (also proposed in [12]). The latter device is briefly discussed in section 5 around equation (20). We also show that counterposition does not satisfy the formal classical criterion of negative refraction which was utilized in the arguments of [6].

The results reported below pertain to electromagnetically homogeneous media. In practice, however, strongly anisotropic crystals are manufactured artificially as layered composites. Therefore, in section 2, we summarize the homogenization procedure for a two-component layered medium. It should be stressed that most (although, not all) of the formulae reported in section 2 are known. Homogenization of a generic two-component layered structure with both electric and magnetic properties was described by Rytov in 1955 [13];

alternating metal–dielectric layers in the electrostatic limit were considered by Tamm and Ginsburg as early as in 1943 [14]. More recently, the problem was considered in great detail for non-magnetic layers by Yeh and Yariv [15, 16] and further by Vinogradov and Dorofeenko [17], and Li *et al* [18]. Finally, the homogenization limit for chiral layers was recently obtained by Ramakrishna and Lakhtakia [19]. However, to give the reader a succinct view of the problem, we find it useful to write without derivation all the relevant formulae in section 2 below. These formulae are scattered in the references cited above and are often hard to locate. Thus, Rytov has considered the first non-vanishing corrections to the homogenization limit [13] (which is not done in other references), but only for propagation directions along or perpendicular to the optical axis. Since the direction of propagation in [13] was fixed, Rytov did not impose the condition that the effective parameters should be independent of the angle of incidence—an important consideration in the theory of homogenization. In the work of Yeh and Yariv [15, 16], magnetic properties were not considered. Finally, we are not aware of any published results for the participation ratio of the fundamental Bloch mode which requires a rather tedious calculation.

Having discussed the homogenization limit in section 2, we consider refraction of narrow Gaussian beams into strongly anisotropic crystals in section 3. Here we use the theory of paraxial beams (rather than the more commonly used approach which relies on the direction of the Poynting vector) in order to obtain information on the optical phase of the refracted beam. The main results of this paper follow in sections 4–6. We argue that the negative deflection of a beam which can be observed in non-magnetic anisotropic media should not be classified as negative refraction. In particular, we discuss the optical phase of the refracted beam in section 4 and the coefficient of transmission through a finite slab in section 5. In section 6, we consider in detail the condition under which *evanescent* waves can experience true negative refraction in anisotropic crystals and show that these conditions are in full agreement with the theory of [6]. Finally, in section 7, we discuss the general suitability of anisotropic media for near-field imaging applications.

2. Homogenization of a planar layered medium: a brief review

As was mentioned above, many of the formulae reported below, including the expressions for the effective parameters, can be found in [13–19], and are given here without derivation for the reader's convenience. We are, however, not aware of any published results for corrections to the homogenization limit (except, in a limited sense, in [13]), or for the participation ratio of the fundamental Bloch mode, all of which are reported below.

Suppose that a two-component layered medium occupies the half-space $z > 0$ and that the wavevector of an incident monochromatic wave lies in the XZ plane (see figure 1). Then the electric and magnetic fields in the region $z > 0$ are Bloch

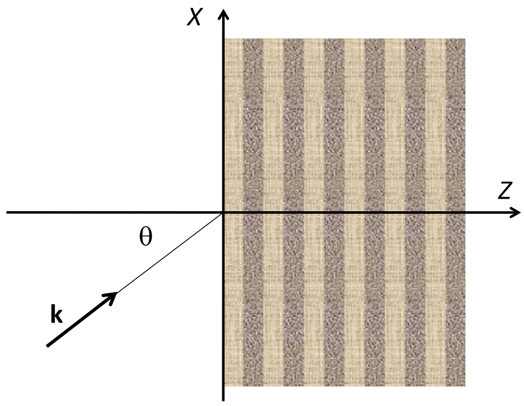

Figure 1. Illustrating geometry of the problem.

waves whose spatial dependence is governed by the expression

$$\exp[\mathrm{i}(k_x x + qz)] \sum_n \mathbf{A}_n \exp(\mathrm{i}g_n z). \qquad (1)$$

Here $k_x > 0$ is the projection of the incident wavevector onto the X-axis, q is the (generally, complex) Bloch wavenumber, $\mathbf{A}_n$ ($n = 0, \pm1, \pm2, \ldots$) are vector amplitudes, $g_n = 2\pi n/h$ are reciprocal lattice vectors and h is the period of the lattice. The Bloch wavenumber q and the field amplitudes $\mathbf{A}_n$ can be obtained by applying the boundary conditions for the fields at all surfaces of discontinuity.

Let us assume that the composite consists of alternating layers with permittivities ϵ_a, ϵ_b, permeabilities μ_a, μ_b and widths a, b such that $a+b = h$. We will use the dimensionless quantities p_a, p_b defined by

$$p_a = a/h, \qquad p_b = b/h, \qquad p_a + p_b = 1. \qquad (2)$$

In order to treat different polarizations of the incident wave on the same footing, we will also use the following notation:

$$\eta_{a,b} = \begin{cases} \mu_{a,b} & \text{(s polarization)} \\ \epsilon_{a,b} & \text{(p polarization).} \end{cases}$$

Here s (transverse electric) polarization corresponds to the wave whose electric field is perpendicular to the plane of incidence (the XZ plane in figure 1) while p (transverse magnetic) polarization corresponds to the magnetic field being perpendicular to the plane of incidence.

The dispersion equation is an expression that couples the Bloch wavenumber q to the frequency ω and other parameters of the problem some of which can also depend on frequency. For the geometry described above, the dispersion equation reads

$$\cos(qh) = \frac{1}{4\mathcal{Z}}[(1+\mathcal{Z})^2\cos(\phi_a+\phi_b) - (1-\mathcal{Z})^2\cos(\phi_a-\phi_b)], \qquad (3)$$

where the following notation has been used:

$$\phi_a = k_{az}a, \qquad \phi_b = k_{bz}b,$$

$$k_{az} = \sqrt{k_0^2\epsilon_a\mu_a - k_x^2}, \qquad k_{bz} = \sqrt{k_0^2\epsilon_b\mu_b - k_x^2},$$

$$\mathcal{Z} = k_{az}\eta_b/k_{bz}\eta_a, \qquad k_0 = \omega/c.$$

Here and everywhere below we use the convention according to which the branch of a square root of an arbitrary complex number z is uniquely determined by the condition $0 \leqslant \arg(\sqrt{z}) < \pi$.

Equation (3) can be solved for q in terms of elementary functions by applying the inverse cosine. The branch of the arccosine is uniquely defined by the conditions $\mathrm{Im}\, q > 0$ and $-\pi/h < \mathrm{Re}\, q \leqslant \pi/h$. It is more instructive, however, to expand the solution in powers of the lattice period, h. Here we assume that the ratios $p_{a,b}$ defined in (2) are fixed and view the layers widths as functions of h: $a = p_a h$, $b = p_b h$. The expansion then reads

$$q = q_0 + \frac{(p_a p_b)^2}{24}\frac{q_1^4}{q_0}h^2 + \mathrm{O}(h^4), \tag{4}$$

where

$$q_0 = \sqrt{k_0^2\langle\epsilon\rangle\langle\mu\rangle - k_x^2\langle\eta\rangle\langle 1/\eta\rangle}, \tag{5a}$$

$$q_1 = \sqrt{\pm k_0^2(\epsilon_a\mu_b - \epsilon_b\mu_a) - k_x^2\left(\frac{\eta_b}{\eta_a} - \frac{\eta_a}{\eta_b}\right)}. \tag{5b}$$

Here '+' in equation (5b) corresponds to s polarization, '−' to p polarization and $\langle\cdots\rangle$ in equation (5a) denotes the cell average. For example,

$$\langle\epsilon\rangle = p_a\epsilon_a + p_b\epsilon_b, \qquad \langle 1/\epsilon\rangle = p_a/\epsilon_a + p_b/\epsilon_b, \text{ etc.}$$

Formula (4) gives the first two terms in the formal Taylor series expansion of the function $q(h)$. If this function is analytic at $h = 0$, the expansion is guaranteed to converge in some vicinity of this point. However, neither the radius of convergence nor an estimate of the reminder can be simply stated. This is because the problem has two physically and mathematically independent parameters of the dimensionality of inverse length: k_0 and k_x. In practice, the accuracy of any approximation involving a finite number of terms can be evaluated by numerical comparison with the exact function $q(h)$ obtained by inverting equation (3). The next term in expansion (4), and also in the similar expansion (10) below, is given in the appendix. Finally, note that expansion (4) is invalid if $q_0 = 0$. The function $q(h)$ is in this case non-analytic at $h = 0$ and behaves as $\sqrt{h}$.

To establish the existence of the homogenization limit and to determine the effective parameters of the layered medium, consider an analogous dispersion equation in a homogeneous uniaxial crystal. If the optical axis of the crystal is aligned with the Z-axis, the elements of the permittivity tensor are $\epsilon_{xx} = \epsilon_{yy} = \epsilon_\parallel$, $\epsilon_{zz} = \epsilon_\perp$, $\epsilon_{xy} = \epsilon_{xz} = \epsilon_{yz} = 0$, and similarly for the permeability. The dispersion equation in the crystal reads [20]

$$k_{tz} = \sqrt{k_0^2\epsilon_\parallel\mu_\parallel - k_x^2\eta_\parallel/\eta_\perp}. \tag{6}$$

Here k_{tz} is the Z-component of the transmitted wavevector and, as above, the variable η denotes permeability in the case of s polarization and permittivity in the case of p polarization. The layered medium can be viewed as 'electromagnetically similar' to a homogeneous crystal if it is possible to find such values of $\epsilon_\parallel$, $\epsilon_\perp$, $\mu_\parallel$, $\mu_\perp$ that the quantities q and k_{tz} coincide with a given precision for all values of k_x (in practice, in a sufficiently large interval of k_x).

We note immediately that the functional equality $q(k_x) = k_{tz}(k_x)$ can hold only if we neglect the $\mathrm{O}(h^2)$ term in expansion (4).[4] Thus we can now rigorously define the homogenization limit. Namely, this limit is an approximation in which the terms of the order of $\mathrm{O}(h^2)$ are neglected in power series expansions of all *physically measurable* quantities. Note that the latter qualification is important, as will be shown below.

If we do neglect the $\mathrm{O}(h^2)$ term in (4), the effective parameters are obtained from the equation $q_0(k_x) = k_{tz}(k_x)$ which must hold for all k_x. The effective parameters which satisfy this equation are

$$\epsilon_\parallel = \beta\langle\epsilon\rangle, \qquad \mu_\parallel = \frac{1}{\beta}\langle\mu\rangle,$$
$$\epsilon_\perp = \beta\left\langle\frac{1}{\epsilon}\right\rangle^{-1}, \qquad \mu_\perp = \frac{1}{\beta}\left\langle\frac{1}{\mu}\right\rangle^{-1}, \tag{7}$$

where β is an arbitrary complex number. Thus, the solution is not unique. It will, however, become clear shortly that the correct choice of effective parameters corresponds to $\beta = 1$.

Let us assume that the first layer in the medium (the one which is adjacent to vacuum) is of the a type. Then the reflection coefficient for the transverse component of the field (E_y for s polarization or H_y for p polarization) is given by

$$r_{\mathrm{LM}} = \frac{k_{iz} - \kappa/\eta_a}{k_{iz} + \kappa/\eta_a}. \tag{8}$$

Here the subscript 'LM' is used to emphasize that this formula is applicable to a layered medium,

$$k_{iz} = \sqrt{k_0^2 - k_x^2} \tag{9}$$

is the Z-axis projection of the incident wavevector and

$$\kappa = k_{az}\frac{1+F}{1-F}, \qquad F = \frac{1+\mathcal{Z}}{1-\mathcal{Z}}\frac{\mathrm{e}^{\mathrm{i}\phi_a} - \mathrm{e}^{\mathrm{i}(qh-\phi_b)}}{\mathrm{e}^{-\mathrm{i}\phi_a} - \mathrm{e}^{\mathrm{i}(qh-\phi_b)}}.$$

Again, we expand the quantity κ defined above in a power series in h (keeping in mind that q is also a function of h) and obtain

$$\kappa = \frac{\eta_a}{\langle\eta\rangle}\left[q_0 - \frac{\mathrm{i}p_a p_b}{2}q_1^2 h + \mathrm{O}(h^2)\right]. \tag{10}$$

Now compare the result given in equations (8) and (10) to the analogous expression for the reflection coefficient of a homogeneous uniaxial crystal, r_{F}. An expression for the transmission coefficient, t_{F}, is also adduced below for later

[4] It is possible to satisfy the functional equality more generally if we allow the effective parameters of the medium to depend explicitly on k_x. Physically, such dependence can be understood as spatial dispersion. This approach, however, is hardly practical since the use of nonlocal effective parameters provides no mathematical simplification or physical insight compared to the approach in which the exact k_x-dependent transmission and reflection coefficients of the medium are used [21].

reference. The quantities r_F, t_F are given by the well-known Fresnel formulae (hence the subscript 'F') and read

$$r_F = \frac{k_{iz} - k_{tz}/\eta_\parallel}{k_{iz} + k_{tz}/\eta_\parallel}, \qquad t_F = \frac{2k_{iz}}{k_{iz} + k_{tz}/\eta_\parallel}. \qquad (11)$$

In the homogenization limit, reflection from a layered medium and a homogeneous crystal should be physically indistinguishable. Hence we require the functional equality $r_{LM}(k_x) = r_F(k_x)$ or, equivalently, $\kappa(k_x)/\eta_a = k_{tz}(k_x)/\eta_\parallel$. Again, it is apparent that this can hold only if we neglect the $O(h)$ term in expansion (10). Note that this corresponds to neglecting the $O(h^2)$ term in the h-expansion of the reflection coefficient r_{LM}. As was mentioned above, the homogenization limit corresponds to neglecting the terms of the order of $O(h^2)$ in expansions of physically measurable quantities such as q or r_{LM} while the variable κ is not directly measurable. After neglecting the $O(h)$ term in (10), we obtain the equation $q_0/\langle\eta\rangle = k_{tz}/\eta_\parallel$. We have already established that, in the homogenization limit, $k_{tz}(k_x) = q_0(k_x)$. Therefore, the previous equation implies that $\langle\eta\rangle = \eta_\parallel$. This condition sets $\beta = 1$ in equations (7). Equation (7) with $\beta = 1$ has been known for a long time (see [13, 15, 16], and for the case of bianisotropic media [19]).

It is also possible to show that, in the homogenization limit, the Bloch wave (1) converges in the L^2 norm to a plane wave. This statement, however, applies only to transverse components of the fields. The Z-components do not converge to a plane wave in the L^2 norm. This indicates that the homogenization procedure described above is, in fact, of limited applicability. Thus, the effective parameters defined in equations (7) would not properly characterize the medium if it was shaped differently from a plane-parallel slab. In other words, if an arbitrarily shaped object is carved from the layered medium described here, it would not be characterizable by the derived effective parameters. This is expected because the medium has small-scale variations in only one spatial direction. In the case of a periodic 3D medium with fine-scale variations in all three directions, the L^2 convergence can be demonstrated quite generally using the multi-scale approach [22].

Returning to the problem at hand, consider the participation ratio

$$\mathcal{P} = \frac{|\hat{\mathbf{y}} \cdot \mathbf{A}_0|^2}{\sum_n |\hat{\mathbf{y}} \cdot \mathbf{A}_n|^2}.$$

Here $\hat{\mathbf{y}}$ is the unit vector in the direction of the Y-axis and $\mathbf{A}_n$ are the Bloch amplitudes for the electric (s polarization) or magnetic (p polarization) fields. A tedious but rather straightforward calculation results in the following expansion:

$$\mathcal{P} = 1 - \frac{(p_a p_b)^2}{12}(h|q_0|)^2 \left|\frac{\eta_a - \eta_b}{\langle\eta\rangle}\right|^2 + O(h^3).$$

Thus, the fundamental Bloch mode dominates the higher-frequency spatial harmonics in the homogenization limit. The first non-vanishing correction is of the order of $O(h^2)$. However, if it so happens that $\eta_a = \eta_b$, this correction vanishes. This happens, for example, in the case of s polarization if the layers are non-magnetic, and so $\eta_a = \eta_b = 1$. In this case, the first non-vanishing correction is of the order of $O(h^3)$ (this correction has not been explicitly computed here).

At this point the following conclusions can be drawn:

(i) In the homogenization limit, electric and magnetic properties do not mix. Specifically, if one starts with non-magnetic layers with $\mu_a = \mu_b = 1$, the effective medium is also non-magnetic with $\mu_\parallel = \mu_\perp = 1$. A similar result applicable to 3D periodic media was derived in [22] by means of the multi-scale analysis.

(ii) In the case of non-magnetic layers, it is possible to obtain a birefringent effective medium with Re $\epsilon_\parallel$ and Re $\epsilon_\perp$ of opposite signs. This can be achieved by using a combination of conducting and dielectric layers and manipulating parameters in equations (7) (with $\beta = 1$). Very strong anisotropy of this kind is not observed in natural crystals.

(iii) The medium can be viewed as electromagnetically homogeneous only for a finite range of k_x. Indeed, when k_x is increased, the corrections in formulae (4) and (10) experience an unbounded growth. At the same time, the homogenization limit is a valid approximation only when these corrections can be neglected.

(iv) In the layered medium considered, the macroscopic Maxwell's equations with effective medium parameters defined by (7) (with $\beta = 1$) are accurate with the precision $O(h^2)$. Taking account of effects which are of this order of magnitude within the macroscopic theory amounts to exceeding the precision of the underlying approximation.

(v) Finally, the homogenization procedure described here is applicable only as long as the medium is a slab. It is not correct to use the derived effective medium parameters if the medium is carved to take any other shape while true effective parameters should be independent of shape.

3. Refraction of Gaussian beams in anisotropic media

We now assume that the medium in question is electromagnetically homogeneous and completely described by the elements of the permittivity tensor $\epsilon_\parallel$, $\epsilon_\perp$. As was shown in section 2, a layered medium made of non-magnetic layers is also non-magnetic in the homogenization limit. Since there is no significant natural magnetism at the frequencies of interest, we assume for now that $\mu_\parallel = \mu_\perp = 1$. As is well-known, *ordinary* (s polarized) waves do not feel the effects of anisotropy in non-magnetic crystals. Therefore, we consider refraction of a monochromatic *extraordinary* (p polarized) Gaussian beam at a planar interface shown in figure 1.

Refraction of normally incident Gaussian beams into a uniaxial crystal whose optical axis is at an arbitrary angle to the interface was described by Stamnes and Dhayalan [23]. We will use below the concept of a two-dimensional Gaussian beam which was introduced in this reference. However, we work in a slightly different geometry: a beam is incident at an arbitrary angle at the interface but the optical axis is orthogonal to the latter. We will be interested, in particular, in the optical phase of the transmitted beam. Therefore, we will explicitly compute the field of the transmitted beam rather than rely on a simpler consideration which is based on the direction of energy transport and which was used, for example, in [4].

Let the incident magnetic field be given by the superposition

$$\mathbf{H}_{\rm i}(x,z) = \hat{\mathbf{y}} \int_{-\infty}^{\infty} a(k_x) \times \exp\{{\rm i}[k_x(x-x_0) + k_{\rm iz}(z-z_0)]\}\frac{{\rm d}k_x}{2\pi}, \qquad (12)$$

where

$$a(k_x) = \frac{\sqrt{\pi}wH_0}{\cos\theta_{\rm i}}\exp\left[-w^2\left(\frac{k_x - \bar{k}_x}{2\cos\theta_{\rm i}}\right)^2\right],$$

$k_{\rm iz}$ is given in (9) and

$$\bar{k}_x = k_0 \sin\theta_{\rm i}.$$

Here x_0 and z_0 are the coordinates of the beam waist (assume that $z_0 < 0$ so that the waist is in vacuum), w is the waist radius, H_0 is the magnetic field amplitude in the waist, and $\theta_{\rm i}$ is the angle of incidence. Note that the quantities $k_{\rm iz}$ and $k_{\rm tz}$ are functions of k_x, but, in order to simplify the notation, this dependence is not indicated explicitly in the Gaussian integral (12) and in similar integrals (13) and (14) (written below).

It is easy to see that, under the condition $w \gg \lambda_0 = 2\pi/k_0$, equation (12) describes a narrow stripe of light which is incident at the interface at the angle $\theta_{\rm i}$. The magnetic field $\mathbf{H}_{\rm i}(x,z)$ is independent of y so that the beam is two dimensional. It is not difficult to consider a truly three-dimensional beam as well, but the magnetic field cannot be strictly polarized along the Y-axis in this case, while all the relevant physics is fully manifest already in the 2D case.

The transmitted and reflected waves, $\mathbf{H}_{\rm t}$ and $\mathbf{H}_{\rm r}$, are found from the boundary conditions and have the form

$$\mathbf{H}_{\rm t}(x,z) = \hat{\mathbf{y}} \int_{-\infty}^{\infty} a(k_x) t_{\rm F}(k_x) \times \exp\{{\rm i}[k_x(x-x_0) + k_{\rm tz}z - k_{\rm iz}z_0]\}\frac{{\rm d}k_x}{2\pi}, \qquad (13)$$

$$\mathbf{H}_{\rm r}(x,z) = \hat{\mathbf{y}} \int_{-\infty}^{\infty} a(k_x) r_{\rm F}(k_x) \times \exp\{{\rm i}[k_x(x-x_0) - k_{\rm iz}(z+z_0)]\}\frac{{\rm d}k_x}{2\pi}, \qquad (14)$$

where the reflection and the transmission coefficients, $r_{\rm F}(k_x)$ and $t_{\rm F}(k_x)$, are given in (11) and $k_{\rm tz}$ is given in (6) (with $\mu_\parallel = \mu_\perp = 1$).

Consider the refracted wave in more detail now. As is usually done in the theory of Gaussian beams [24], we expand the functions $k_{\rm iz}(k_x)$, $k_{\rm tz}(k_x)$ to the first order in Taylor series near the point $k_x = \bar{k}_x$. This yields

$$\mathbf{H}_t(x,z) \approx \hat{\mathbf{y}}\exp\{{\rm i}[\bar{k}_x(x-x_0) + \bar{k}_{\rm tz}z - \bar{k}_{\rm iz}z_0]\} \times \int_{-\infty}^{\infty} a(k_x)t_{\rm F}(k_x)\frac{{\rm d}k_x}{2\pi} \times \exp\{{\rm i}(k_x - \bar{k}_x)[x - x_1 - (\bar{k}_x\epsilon_\parallel/\bar{k}_{\rm tz}\epsilon_\perp)z]\}. \qquad (15)$$

Here $\bar{k}_{\rm iz} = k_{\rm iz}(\bar{k}_x)$, $\bar{k}_{\rm tz} = k_{\rm tz}(\bar{k}_x)$ and $x_1 = x_0 - (\bar{k}_x/\bar{k}_{\rm iz})z_0$ is the X-coordinate of the intersection of the incident beam with the interface (recall that $z_0 < 0$) and the approximation is valid for not very large values of z.

From equation (15), it follows that the amplitude of the refracted magnetic field can be approximately written as a function of the variable $\xi = x - x_1 - (\bar{k}_x\epsilon_\parallel/\bar{k}_{\rm tz}\epsilon_\perp)z$. In particular, for weakly absorbing media, when $\epsilon_\parallel$, $\epsilon_\perp$ can be approximately viewed as real quantities, the axis of the refracted beam can be found from the equation $\xi = 0$ or, equivalently,

$$x = x_1 + (\bar{k}_x\epsilon_\parallel/\bar{k}_{\rm tz}\epsilon_\perp)z. \qquad (16)$$

In isotropic media $\epsilon_\parallel = \epsilon_\perp$ and refraction of Gaussian beams occurs at the same angle as refraction of plane waves. However, in anisotropic media the situation is different. A plane wave with $k_x = \bar{k}_x$ is refracted at the angle

$$\theta_{\rm t}^{\rm (p.w.)} = \arctan(\bar{k}_x/\bar{k}_{\rm tz}),$$

which is Snell's law. But a Gaussian beam is refracted at the angle

$$\theta_{\rm t}^{\rm (G.b.)} = \arctan(\bar{k}_x\epsilon_\parallel/\bar{k}_{\rm tz}\epsilon_\perp).$$

If it so happens that $\epsilon_\parallel$ and $\epsilon_\perp$ have opposite signs (and it is still possible to neglect absorption), refraction of the Gaussian beam occurs at a negative angle.

Note that the above calculation is quite independent of the expression for the Poynting vector which is frequently invoked to determine the direction of a beam refracted into an anisotropic medium. Generally speaking, as long as the correct boundary condition at the infinity is used (the so-called Sommerfeld radiation condition), the Maxwell equations can be solved and the fields can be found without any knowledge of the Poynting vector.

As was mentioned in section 2 (conclusion (ii)), crystals with $\epsilon_\parallel$ and $\epsilon_\perp$ of different sign do not occur naturally but can be manufactured. For example, a particular design was suggested in [4]. In this reference, alternating 40 nm thick planar layers of InGaAs and AlInAs were grown by molecular beam epitaxy to create a 8 μm thick layered film. The InGaAs layers were doped to provide a controllable concentration of free charge carriers. It was estimated that the permittivity of AlInAs layers was approximately constant in the far-IR spectral range and equal to $\epsilon_a = 10.23$ while the permittivity of InGaAs was given by a Drude-like formula $\epsilon_b = 12.15 - \omega_{\rm p}^2/\omega(\omega + {\rm i}\gamma)$. The experimental parameters in the above formula were as follows: the vacuum wavelength at the plasma frequency was $\lambda_{\rm p} = 2\pi c/\omega_{\rm p} = 2.53$ μm and the ratio of the Drude relaxation constant to the plasma frequency was $\gamma/\omega_{\rm p} = 0.0134$. This completely defines the permittivities ϵ_a and ϵ_b of the layers. The experiment in [4] was conducted at the frequency ω such that $\lambda_0 = 2\pi c/\omega = 9.5$ μm. This corresponds to $\omega/\omega_{\rm p} = 0.266$ and $\epsilon_b = -1.95 + 0.71{\rm i}$.

For the experimental parameters given above, the medium manufactured in [4] can be viewed as electromagnetically homogeneous with good precision. In particular, it can be verified that the exact Bloch wavenumber q and the first non-vanishing term q_0 in the expansion (4) coincide numerically

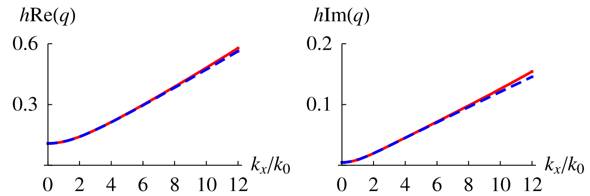

Figure 2. Real and imaginary parts of the Bloch wavenumber q computed exactly by inverting equation (3) (solid red curves) and by keeping only the first non-vanishing term q_0 in expansion (4) (dashed blue curves), plotted as functions of k_x for the experimental parameters described in the text.

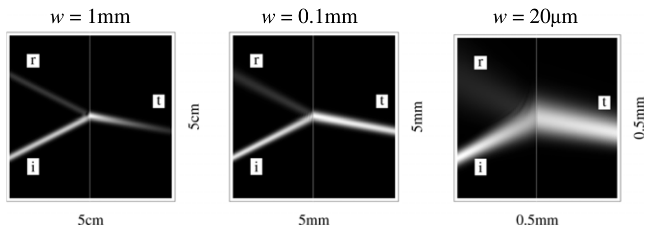

Figure 3. Reflection and refraction of a monochromatic p polarized Gaussian beam of different waist sizes w at the interface with a semi-infinite medium described in the text. The angle of incidence is $\theta_i = \pi/4$. The amplitude of the total magnetic field, $|\mathbf{H}|$ where $\mathbf{H} = \mathbf{H}_i + \mathbf{H}_r + \mathbf{H}_t$, is shown by a linear grayscale. White color corresponds to $|\mathbf{H}| = H_0$ where H_0 is the magnetic field intensity in the waist. The incident beam is marked by the letter 'i', the reflected beam by the letter 'r' and the transmitted beam by the letter 't'.

with high accuracy in the interval $k_x \lesssim 10k_0$. This is illustrated in figure 2. Therefore, the medium reflects and refracts light as a homogeneous uniaxial crystal whose permittivity tensor has the following elements (at the working frequency): $\epsilon_\parallel = 4.14 + 0.36\mathrm{i}$ and $\epsilon_\perp = -4.63 + 2.15\mathrm{i}$.

Of course, the presence of free charge carriers in one of the layers results in substantial losses as can be seen from the numbers adduced above. Therefore, it is possible to talk about refraction of beams into the experimental medium of [4] only with a certain amount of strain. However, this is not a point of principle. In figure 3, we illustrate refraction of Gaussian beams into a medium which is similar in all respects to the medium of [4] but with substantially smaller losses. Specifically, we have reduced the experimental value of the ratio $\gamma/\omega_p = 0.0134$ by a factor 10^3. This resulted in approximately the same real parts of the effective medium parameters as in [4] but in much smaller imaginary parts: $\epsilon_\parallel = 4.12 + 3.56 \times 10^{-4}\mathrm{i}$ and $\epsilon_\perp = -4.92 + 2.19 \times 10^{-3}\mathrm{i}$. Note that, although these parameters are not achievable experimentally (at least, at the present time), we did not violate any of the fundamental physical principles such as causality. The intensity of the magnetic field was obtained by numerical integration according to (12)–(14) without any additional approximations. As can be seen from the figure, narrow Gaussian beams can indeed be refracted into the medium at a negative angle.

But can this phenomenon be called negative refraction? It may seem that this question is purely a matter of terminology. Besides, a distinction is increasingly being made in the literature between backward waves and waves refracted at a negative angle due to nonsphericity of the isofrequency surface [10, 11]. This can potentially serve to separate the two physically unrelated phenomena. However, the point of view stated above is highly disagreeable. The reason is the enormous body of literature published in the past ten years in which the terms have been used interchangeably and in which negative refraction was associated with extraordinary optical properties and unprecedented applications. At the same time, negative refraction as it was first introduced by Mandelstam [25] and later by Sivukhin [26], and as it was exploited in the now famous paper by Pendry [12], refers specifically to negative index of refraction and does not include the beam deflection phenomenon described and simulated above.

In the next two sections, we discuss the fundamental physical differences between classical negative refraction and negative deflection of a beam. At this point, however, it is worth mentioning that there are many different means to force a beam to be deflected at an interface in any given direction, including the use of anisotropy, diffraction gratings or even an array of micromirrors. However, none of these techniques can be used to solve any of the fundamental problems in optics such as overcoming the diffraction limit of resolution in an imaging system.

4. The optical phase

As was noted in [4], the Gaussian wavepackets shown in figure 3 are superpositions of plane waves which experience ordinary (positive) refraction at the interface. This follows from the inequality $\mathrm{Im}\, k_{tz}^2 > 0$ which holds for the medium of [4] (and for the medium used above for simulations) for all values of k_x. The inequality $\mathrm{Im}\, k_{tz}^2 < 0$ as the condition of true negative refraction is discussed in more detail in section 6. We can conclude that the optical phase of the refracted beam increases in the direction of the Z-axis. Indeed, it follows from (15) and (16) that the optical phase at the refracted beam axis is

$$\phi = \phi_0 + (k_0^2 \epsilon_\parallel / \bar{k}_{tz}) z. \quad (17)$$

In this expression, ϕ_0 is the optical phase at the point of ray intersection with the interface while z should be understood as the Z-coordinate of an arbitrary point on the refracted beam axis. Thus, the second term on the right-hand side of (17) is the optical phase associated with beam propagation. Here we still assume that losses are small so that $\epsilon_\parallel$ can be viewed as a positive number. It is therefore apparent that the optical phase increases with z. This is one of the properties that distinguish a Gaussian beam refracted at a negative angle in a medium described in section 3 from waves experiencing true negative refraction. As is well-known, the latter are backward waves whose vector of phase velocity points in the direction of the interface rather than away from it.

To conclude this section, note that the coefficient in the second term in the right-hand side of (17) has a physical meaning: it is related to the rate at which the medium is heated by the radiation (more precisely, the quantity in question is $\mathrm{Re}(\epsilon_\parallel / k_{tz})$). This will be shown in section 6. Correspondingly, this coefficient cannot be negative as long as $\epsilon_\parallel$ and $\epsilon_\perp$ have positive imaginary parts.

5. Transmission through a slab

We now come to the central and most practically relevant physical characteristic which distinguishes anisotropic non-magnetic crystals from media with negative refraction. Namely, non-magnetic crystals cannot exponentially amplify the transmission coefficient for evanescent waves, except in the limited sense as discussed below.

Consider the coefficient of transmission T through an anisotropic uniaxial slab of width L (not to be confused with the Fresnel transmission coefficient at a single interface, t_F). We do not restrict attention to non-magnetic crystals in this section and allow both electric and magnetic anisotropy with the same optical axis which points in the Z-direction. Then the transmission coefficient for transverse components of the field (E_y for s polarization and H_y for p polarization) is given by

$$T = \frac{4pC}{(C+1)^2 - p^2(C-1)^2},$$

where

$$p = \exp(\mathrm{i}k_{tz}L), \qquad C = \frac{k_{iz}\eta_\parallel}{k_{tz}}.$$

Here k_{tz}, k_{iz} are given by formulae (6) and (9).

First, we note that for waves which are propagating in vacuum (that is, for waves with $k_x < k_0$), the transmission coefficient is bounded by $|T| \leqslant 1$. This follows immediately from energy conservation. However, for waves with $k_x > k_0$, this consideration does not apply because evanescent waves in vacuum do not transport energy[5]. It is therefore possible to have $|T| > 1$ if $k_x > k_0$. It can be seen that T can exponentially grow with L if and only if it turns out that, in some range of k_x (contained in the interval $k_x > k_0$), $C = -1$. In this case, $T \propto 1/p = \exp(-\mathrm{i}k_{tz}L)$. Further, it is not difficult to see that the condition $C = -1$ is satisfied for all $k_x > k_0$ if

$$\epsilon_\parallel = \mu_\parallel = 1/\eta_\perp < 0. \qquad (18)$$

Here η refers to permeability for s polarization, permittivity for p polarization and the condition (18) must be applied to each polarization separately. If, in addition to (18) holding, it turns out that $\mu_\perp = \epsilon_\perp$, then exponential amplification of the transmission coefficient occurs independently of polarization. In the isotropic case, equation (18) simplifies to the well-known condition $\epsilon = \mu = -1$. In practice, of course, these conditions can be satisfied only approximately because $\epsilon_\parallel$, $\epsilon_\perp$, $\mu_\parallel$, $\mu_\perp$ have nonzero (and, arguably, quite large [27]) imaginary parts. The more precise condition for the exponential amplification of the transmission coefficient T can be stated as

$$\left|\frac{C+1}{C-1}\right| \ll |p|^2 \ll 1. \qquad (19)$$

The conclusion that one can draw from the expressions given above is the following. In magnetic anisotropic crystals, the condition (18) can be satisfied with any given precision if we view the variables $\epsilon_\parallel$, $\epsilon_\perp$, $\mu_\parallel$, $\mu_\perp$ as mathematically independent free parameters with the sole restriction that their imaginary parts must be positive. This approach does not account for additional constraints on the susceptibilities of [6] and is, therefore, unphysical. Still, from the purely formal, mathematical point of view, the approach is not incorrect. However, in non-magnetic crystals, we have $\mu_\parallel = \mu_\perp = 1$ and the condition (18) cannot be satisfied in principle, even approximately. Consequently, the negative deflection of a beam at the boundary of a non-magnetic uniaxial medium cannot and should not be classified as negative refraction because this phenomenon lacks the single most practically relevant property of the latter: the exponential amplification of T.

Note however that (18) is a sufficient but not a necessary condition for $C = -1$. Even if (18) does not hold, it is possible to achieve $C = -1$ and, consequently, exponential amplification of T for a single value of k_x which corresponds to the wavenumber of the surface plasmon polariton (SPP). Subject to additional conditions imposed below, this value is

$$k_x^2 = \pm k_0^2 \frac{\mu_\parallel - \epsilon_\parallel}{\eta_\parallel - 1/\eta_\perp}, \qquad \text{if } \eta_\parallel < 0. \qquad (20)$$

Here '+' corresponds to s polarization, '−' to p polarization and, as before, the formula must be applied to each polarization separately. In order for equation (20) to be applicable, it is additionally required that the result obtained from this formula satisfies $k_x > k_0$. In the case of isotropic non-magnetic media such as ordinary metals, an s polarized SPP does not exist, while for p polarization, equation (20) simplifies to the well-known SPP dispersion equation $k_x^2 = k_0^2\epsilon/(\epsilon + 1)$ (provided that $\epsilon \leqslant -1$). However, exponential growth of T for a single value of k_x cannot be used in imaging applications. This restriction can be alleviated to a certain degree by noting that, if $\epsilon = -1$ and $k_x/k_0 \gg 1$, then, to the first non-vanishing order in k_0/k_x, $C \approx -1 + (k_0/k_x)^2$. Substituting this result into (19), we find that, for a given thickness of the slab, amplification of the transmission coefficient T is achieved as long as $(k_0/k_x)^2 \ll |p| \ll 1$. This is the basis of operation of the so-called 'hyperlens' which was first proposed in [12] and requires neither negative refraction nor anisotropy.

6. True negative refraction of evanescent waves

The physics and history of negative refraction have been amply reviewed by Boardman [11], Agranovich and Gartstein [28] and by Rautian [29]. In these three reviews the focus is on the Fresnel reflection of plane waves. It is shown that the wavevector of a negatively refracted wave has the following property: the real and imaginary parts of its projection onto the Z-axis have opposite signs. This condition can be mathematically stated as $\mathrm{Im}\, k_{tz}^2 = 2\,\mathrm{Re}\, k_{tz}\, \mathrm{Im}\, k_{tz} < 0$. The corresponding diagram has been published many times and there is no need to reproduce it here. The relation between the above condition and the conditions based on the directions of phase and group velocities and on the direction of energy flow are also discussed in these references and, in even greater detail, in the review by Shevchenko [30]. It can be stated that the criterion $\mathrm{Im}\, k_{tz}^2 < 0$ is the most general and directly usable.

[5] To be more precise, we mean here transport of energy in the direction of the Z-axis. An evanescent wave can still transport energy in the X-direction.

It does not require any knowledge of the dispersion law for the susceptibilities and is applicable even in the presence of substantial losses. The forthcoming discussion is based on the premise (which is substantiated in the above four reviews) that true negative refraction at a planar interface occurs if and only if $\mathrm{Im}\, k_{tz}^2 < 0$.

In the recent paper [6], one of the authors has derived an expression for the heating rate (the amount of heat absorbed by the medium per unit time per unit volume) due to a plane monochromatic wave with the wavevector $\mathbf{k}$ and the amplitude of the electric field $\mathbf{E}_0$ propagating in a general anisotropic, nonlocal, electrically and magnetically polarizable medium (equation (54) in the above reference). The expression is

$$q_V = \frac{\omega\, \mathrm{e}^{-2\mathbf{k}''\cdot\mathbf{r}}}{8\pi k_0^2}\mathrm{Im}[|\mathbf{E}_0|^2(\mathbf{k}\cdot\mathbf{k}) - (\mathbf{k}\cdot\mathbf{E}_0)(\mathbf{k}\cdot\mathbf{E}_0^*)]. \quad (21)$$

Here the double prime indicates imaginary part and the subscript 'V' is used to indicate that this expression is applicable to the points inside the medium. At the surface, there is an additional term q_S which is of no interest here; besides, in non-magnetic media, the surface term is identically zero. It is then stated on physical grounds that the inequality $q_V > 0$ must hold, at least for waves that do not decay very fast inside the medium. It should be noted that in non-magnetic media, expression (21) is a straightforward generalization of the well-known expression for the heating rate (e.g., see [31]) to the case of nonlocal and/or anisotropic media. Thus, for non-magnetic media which are discussed in this section, the expression (21) is conventional.

To analyze equation (21) and to deduce the physical restrictions that it places on the electromagnetic properties of materials, we need first to make a distinction between evanescent and propagating waves. A general complex wavevector $\mathbf{k}$ can be written as $\mathbf{k} = k_r\hat{\mathbf{u}}_r + k_c\hat{\mathbf{u}}_c$ where $\hat{\mathbf{u}}_r$, $\hat{\mathbf{u}}_c$ are purely real unit vectors such that $\hat{\mathbf{u}}_r\cdot\hat{\mathbf{u}}_r = \hat{\mathbf{u}}_c\cdot\hat{\mathbf{u}}_c = 1$, $\hat{\mathbf{u}}_r\cdot\hat{\mathbf{u}}_c = 0$, k_r is a purely real scalar and k_c is a complex scalar. The expansion is unique up to simultaneous change of signs in the quantities k_r, $\hat{\mathbf{u}}_r$ and/or k_c, $\hat{\mathbf{u}}_c$. Note that an ambiguity does appear if $\mathbf{k}$ is purely real, e.g., for waves in vacuum. The ambiguity is removed by requiring that, in this case, $k_r = 0$ by definition. In material media the vector $\mathbf{k}$ has an imaginary part, however small, which guarantees the expansion uniqueness. We will call a wave propagating if $k_r = 0$ and evanescent otherwise. Thus, a propagating wave can, in principle, experience exponential decay or growth due to a nonzero imaginary part of k_c. However, it is characterized by a single direction in space given by $\hat{\mathbf{u}}_c$. Propagation of an evanescent wave is, in contrast, characterized by two orthogonal directions. It is not possible to tell in which direction an evanescent wave propagates; it is only possible to tell in which direction it decays and in which direction it oscillates.

Now if a wave is propagating, its wavevector is $\mathbf{k} = k_c\hat{\mathbf{u}}_c$ and it can be shown that

$$(\mathbf{k}\cdot\mathbf{E}_0)(\mathbf{k}\cdot\mathbf{E}_0^*) = k_c^2|\mathbf{E}_0|^2\cos^2\vartheta, \quad (22)$$

where ϑ is a purely real angle and $\cos^2\vartheta < 1$. In this case, it follows from (21) that $q_V \propto \sin^2\vartheta\, \mathrm{Im}\, k_c^2 \propto \mathrm{Im}\, k_c^2$. This holds irrespectively of the form of the complex vector $\mathbf{E}_0$. Therefore, negative refraction is prohibited for propagating waves.

Let us now specialize to the planar geometry considered in this paper. The wavevector of the refracted wave is $\mathbf{k} = \hat{\mathbf{x}}k_x + \hat{\mathbf{z}}k_{tz}$. Since k_x is purely real, we immediately identify $\hat{\mathbf{u}}_r = \hat{\mathbf{x}}$, $k_r = k_x$ and $\hat{\mathbf{u}}_c = \hat{\mathbf{z}}$, $k_c = k_{tz}$. A normally incident wave is propagating since it has $k_r = k_x = 0$. For such a wave, we have $q_V \propto \mathrm{Im}\, k_{tz}^2$ and negative refraction is prohibited. For the incidence directions close to the normal, the formula $q_V \propto \mathrm{Im}\, k_{tz}^2$ still holds, albeit approximately. Indeed, the quantity q_V is a continuous function of k_x which is positive at $k_x = 0$ and, consequently, in a finite vicinity of this point. Therefore, negative refraction is not possible for a range of incident directions around the normal.

However, as k_x increases, the refracted wave becomes more and more evanescent. At some point, it can no longer be characterized by a single direction of propagation. Then the formula (22) becomes invalid and it is no longer possible to relate q_V to $\mathrm{Im}\, k_{tz}^2$. Then, conceivably, the latter can become negative.

The above discussion is an expanded recounting of the result briefly stated in section 6 of [6]. The result is that negative refraction in anisotropic media is only possible for evanescent waves. However, we have not addressed so far the role of anisotropy. The latter is quite profound. If the medium is isotropic, then $\mathbf{k}\cdot\mathbf{E}_0 = 0$ and negative refraction is not possible for either propagating or evanescent waves. In anisotropic media, the above condition is replaced by $\mathbf{k}\cdot\hat{\epsilon}\mathbf{E}_0 = 0$ where $\hat{\epsilon}$ is a tensor. Therefore, the factor $\mathbf{k}\cdot\mathbf{E}_0$ does not turn to zero and the expression (21) is proportional to $\mathrm{Im}(\mathbf{k}\cdot\mathbf{k})$ only for propagating waves, as discussed above.

Let us now find the threshold values of k_x for which negative refraction is possible in a non-magnetic uniaxial crystal. To this end, we apply the criterion $\mathrm{Im}\, k_{tz}^2 < 0$ to expression (6). In the latter formula, we take into account the fact that the crystal is non-magnetic. We then immediately conclude that for s polarization, negative refraction is not possible at all and, for p polarization, it is possible for $k_x > k_c$ where

$$k_c^2 = k_0^2\frac{\epsilon_\parallel''}{(\epsilon_\parallel/\epsilon_\perp)''}, \quad (23)$$

provided that the expression in the right-hand side of (23) is positive. Since $\mathrm{Im}\,\epsilon_\parallel > 0$, negative refraction is possible only if $\mathrm{Im}\,(\epsilon_\parallel/\epsilon_\perp) > 0$.

We now further specialize to the case of a layered medium. Let $\epsilon_\parallel$, $\epsilon_\perp$ be the effective medium parameters given by equations (7) (with $\beta = 1$). Writing out the cell averages explicitly, we obtain

$$k_c^2 = k_0^2\frac{p_a\epsilon_a'' + p_b\epsilon_b''}{p_a p_b(1/|\epsilon_b|^2 - 1/|\epsilon_a|^2)(\epsilon_a\epsilon_b^*)''}. \quad (24)$$

Consider two special cases. First, let the layers be both dielectric with very small losses so that $\epsilon_a = R_a\mathrm{e}^{\mathrm{i}\varphi_a}$, $\epsilon_b = R_b\mathrm{e}^{\mathrm{i}\varphi_b}$ where $R_a, R_b \sim 1$ and $0 < \varphi_a, \varphi_b \ll \pi/2$. We then rewrite (24) approximately as

$$k_c^2 \approx k_0^2\frac{p_a R_a\varphi_a + p_b R_b\varphi_b}{p_a p_b(\varphi_a - \varphi_b)(R_a^2 - R_b^2)/R_a R_b}.$$

All other things being equal, the above expression is minimized if $\varphi_b = 0$. Therefore, let us take the b-medium to be vacuum with $R_b = 1$ and $\varphi_b = 0$. The a-medium is a transparent dielectric with $R_a = \epsilon_d > 0$. Note that the phase φ_a will not enter the following equation and is, therefore, unimportant, as long as it is sufficiently small that the approximation adopted is accurate. We then obtain

$$k_c^2 \approx \frac{k_0^2 \epsilon_d^2}{p_v(\epsilon_d^2 - 1)}, \qquad k_{tz}^2(k_x = k_c) \approx -\frac{k_0^2[p_v + p_d\epsilon_d]^2}{p_v(\epsilon_d^2 - 1)},$$

where $p_d = p_a$ and $p_v = p_b$ are the volume fractions of dielectric and vacuum. We thus see that negative refraction is only possible if $\epsilon_d > 1$ (which is typical) and then only for $k_x > k_c > k_0$. For these values of k_x, the wave is evanescent both in vacuum and in the medium.

The second special case is a metal–dielectric composite. For the sake of the argument, we will assume that the losses in the metal are small and that the dielectric is absolutely transparent (with no losses). We then take $\epsilon_a = -|\epsilon_m| e^{-i\varphi_m}$, $\epsilon_b = \epsilon_d$, where $|\epsilon_m| \gg \epsilon_d > 0$, $0 < \varphi_m \ll \pi/2$ and, using the same arguments as above, arrive at

$$k_c^2 \approx \frac{k_0^2 \epsilon_d}{p_d}, \qquad k_{tz}^2(k_x = k_c) \approx -2k_0^2 p_m |\epsilon_m|,$$

where $p_m = p_a$ and $p_d = p_b$ are the volume fractions of the metal and dielectric. If $\epsilon_d > 1$, as is usually the case, negative refraction is possible only for waves with $k_x > k_c > k_0$ which are evanescent both in vacuum and in the medium. If, however, we consider a hypothetical dielectric with $0 < \epsilon_d < 1$, the inequality $k_c > k_0$ may be broken and negative refraction may be obtained for waves which are propagating in vacuum for the values of k_x in the interval $k_c < k_x < k_0$. The transmitted wave, however, is strongly evanescent even in this case since k_{tz}^2 is negative in the above interval of k_x. This is in agreement with the conclusions of [6].

We thus conclude that in non-magnetic uniaxial crystals, negative refraction is only possible for evanescent waves and then only in a formal sense—as discussed in section 5, it cannot result in an exponential amplification of the transmission coefficient.

Next, we simplify the expression (21) for a p polarized wave incident on a uniaxial non-magnetic crystal. The simplification is achieved by taking into account the equation $\nabla \cdot \mathbf{D} = \nabla \cdot \hat{\epsilon}\mathbf{E} = 0$ which, for p polarization, becomes $k_x \epsilon_\parallel E_{0x} + k_{tz}\epsilon_\perp E_{0z} = 0$. A straightforward algebraic manipulation results in

$$q_V = \frac{\omega\, e^{-2k''_{tz} z}}{4\pi} |E_{0x}|^2 k''_{tz} \,\mathrm{Re}\, \frac{\epsilon_\parallel}{k_{tz}}. \tag{25}$$

The quantity k''_{tz} is positive by definition (recall the convention on the square roots stated above). Therefore, positivity of q_V requires that the factor $\mathrm{Re}(\epsilon_\parallel / k_{tz})$ be positive. Indeed, it can be rigorously proved that the latter factor is positively defined for all values of k_x as long as $\epsilon''_\parallel, \epsilon''_\perp > 0$.

What appears to be important here is that the positive factor $\mathrm{Re}(\epsilon_\parallel / k_{tz})$ that enters equation (25) is the same as the coefficient in the expression (17) which gives the rate at which the optical phase of a Gaussian beam increases with z. It turns out that this coefficient is positively defined and that this constraint is related to the physically required positivity of q_V. Thus, even though negative refraction of evanescent waves is possible in the sense described above, any well-defined Gaussian beam refracted in a uniaxial crystal is a forward wave whose optical phase increases with the distance from the interface.

7. Discussion

We have shown that the theoretical results of [6] are not in contradiction with the experimental observations of [3, 4]. This is because negative deflection of a beam by non-magnetic anisotropic crystals should be viewed not as negative refraction but as counterposition. True negative refraction can occur for evanescent waves (in agreement with [6]) but this occurrence does not result in exponential amplification of the transmission coefficient. Therefore, non-magnetic crystals are fundamentally unsuitable for forming an image with subwavelength resolution in the far zone. This result, of course, does not prove the correctness of [6] but, at least, it shows that there is no contradiction between [6] and a series of rather convincing experiments.

Still, strongly anisotropic crystals are fascinating objects with unusual optical properties. Under certain conditions, a wave which is strongly evanescent in vacuum can refract into such crystal as a propagating wave (if absorptive losses can be neglected) [17]. In this respect, anisotropic media are similar to the conventional near-field imaging devices which couple the near field of a sample to propagating modes of optical waveguides such as fibers [32]. The intensity of these propagating modes can be eventually registered by a detector placed in the near-field zone of the opposite face of the crystal [32]. But can this property of strongly anisotropic non-magnetic crystals be used for conventional near-field imaging? Before answering the above question, it is useful to consider the following.

In conventional near-field imaging, it has long been recognized that a map of the near field may have little or no resemblance to the imaged object [33]. First, there is no local relation between the geometrical structure of the sample and its near field. Second, the source of the near field is the current or the polarization, not the susceptibility of the sample, and the spatial distributions of these two quantities can be very different. Third, the near field can be extremely sensitive to the wavelength [34] or the polarization [35]. In summary, there are fundamental differences between near-field imaging and conventional image formation based on the laws of geometrical optics which allow the tracing of each ray to a point on the surface of the sample. Consequently, significant effort has been devoted to developing a methodology for interpreting near-field images [36–41].

The methodology mentioned above depends on two conditions: first, that the sample is passive and illuminated by an external source and, second, that the probe (the near-field microscope tip) is small and weakly perturbing. These two conditions are often violated when anisotropic layered [18, 42, 43] or wire [32] media are proposed for imaging applications. The imaged object is typically an active source (i.e., a radiating antenna) and the imaging device is a crystal which is brought into the near-field vicinity of a much smaller sample. However, if the sample is passive and illuminated by an external source, as is typical in imaging applications, and if the crystal is very close to the sample, the near field can be perturbed to such an extent as to render it useless.

The second factor rarely considered is that, even if one manages to create a perfect map of the experimental near field that exists in the plane $z = z_1$ at some other plane $z = z_2$, this does not really solve the imaging problem. Conventional optical devices such as lenses, microscopes or CCD cameras are still incapable of registering the field in the plane $z = z_2$ without a severe loss of resolution. On the other hand, if a near-field imaging device (such as the conventional near-field microscope) is used in the plane $z = z_2$, the same device could be used directly in the plane $z = z_1$. Therefore, the advantages of using the crystal to translate the field from the plane $z = z_1$ to the plane $z = z_2$ are not clear.

We are now ready to answer the question posed above. Our conclusion is that non-magnetic anisotropic crystals offer no advantages compared to conventional near-field microscopy but many unwanted and image-degrading effects. Among such effects are losses, distortions and severe perturbation of the near field of the sample.

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Appendix. Higher-order terms in expansions (4) and (10)

In equation (4),

$$O(h^4) = \frac{(p_a p_b)^2}{5760} \frac{q_1^4[16q_0^2 q_2^2 - 5(p_a p_b)^2 q_1^4]}{q_0^3} h^4 + O(h^6),$$

where the quantities q_0 and q_1 are given in equations (2) and

$$q_2^2 = k_0^2[2p_a p_b(\epsilon_a \mu_b + \epsilon_b \mu_a) + p_a^2 \epsilon_a \mu_a + p_b^2 \epsilon_b \mu_b] - k_x^2 \left[2p_a p_b \left(\frac{\eta_a}{\eta_b} + \frac{\eta_b}{\eta_a}\right) + p_a^2 + p_b^2\right].$$

In equation (10),

$$O(h^2) = \frac{p_a p_b}{24} \frac{q_1^2 q_3^2}{q_0} h^2 + O(h^3),$$

where

$$q_3^2 = k_0^2[4(p_a^2 \epsilon_a \mu_a - p_b^2 \epsilon_b \mu_b) \pm 3p_a p_b(\mu_a \epsilon_b - \mu_b \epsilon_a)] - k_x^2 \left[3p_a p_b \left(\frac{\eta_a}{\eta_b} - \frac{\eta_b}{\eta_a}\right) + 4(p_a^2 - p_b^2)\right].$$

In the above formula, '+' must be chosen for s polarization and '−' for p polarization.

References

[1] Belov P A 2003 *Microw. Opt. Technol. Lett.* **37** 259
[2] Yao J, Liu Z, Liu Y, Wang Y, Sun C, Bartal G, Stacy A M and Zhang X 2008 *Science* **321** 930
[3] Zhang Y, Fluegel B and Mascarenhas A 2003 *Phys. Rev. Lett.* **91** 157404
[4] Hoffman A J, Alekseyev L, Howard S S, Franz K J, Wasserman D, Podolskiy V A, Narimanov E E, Sivco D L and Gmachl C 2007 *Nat. Photon.* **6** 946
[5] Zhao Q, Kang L, Li B, Zhou J, Tang H and Zhang B 2006 *Appl. Phys. Lett.* **89** 221918
[6] Markel V A 2008 *Opt. Express* **16** 19152
[7] Lakhtakia A and McCall M W 2004 *Optik* **115** 28
[8] He M, Du Y X, Wang W Y, Zhang D F and Chen X L 2007 *Physica* B **399** 121
[9] Mackay T G and Lakhtakia A 2007 *Microw. Opt. Technol. Lett.* **49** 874
[10] Mackay T G and Lakhtakia A 2009 *Phys. Rev.* B **79** 235121
[11] Boardman A D, King N and Velasco L 2005 *Electromagnetics* **25** 365
[12] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[13] Rytov S M 1956 *Sov. Phys.—JETP* **2** 466–75
[14] Tamm I E and Ginzburg V I 1943 *Izv. Akad. Nauk SSSR Ser. Fiz.* **7** 30
[15] Yeh P, Yariv A and Hong C S 1977 *J. Opt. Soc. Am.* **67** 423
[16] Yariv A and Yeh P 1977 *J. Opt. Soc. Am.* **67** 438
[17] Vinogradov A P and Dorofeenko A V 2005 *J. Commun. Technol. Electron.* **50** 1153
[18] Li X, He S and Jin Y 2007 *Phys. Rev.* B **75** 045103
[19] Ramakrishna A A and Lakhtakia A 2009 *J. Opt. A: Pure Appl. Opt.* **11** 074001
[20] Arakelian V H, Bagdassarian L and Simonian S G 1997 *J. Magn. Magn. Mater.* **167** 149
[21] Menzel C, Rockstuhl T P, Paul T, Lederer F and Pertsch T 2008 *Phys. Rev.* B **77** 195328
[22] Wellander N and Kristensson G 2003 *SIAM J. Appl. Math.* **64** 170
[23] Stamnes J J and Dhayalan V 2001 *J. Opt. Soc. Am.* A **18** 1662
[24] Born M and Wolf E 1999 *Principles of Optics* (Cambridge: Cambridge University Press)
[25] Mandelstam L I 1945 *Zh. Eksp. Teor. Fiz.* **15** 475
[26] Sivukhin D V 1957 *Opt. Spectrosc.* **3** 308
[27] Stockman M I 2007 *Phys. Rev. Lett.* **98** 177404
[28] Agranovich V M and Gartstein Y N 2006 *Phys.—Usp.* **49** 1029
[29] Rautian S G 2008 *Phys.—Usp.* **51** 981
[30] Shevchenko V V 2007 *Phys.—Usp.* **50** 287
[31] Landau L D and Lifshitz L P 1984 *Electrodynamics of Continuous Media* (Oxford: Pergamon) sec. 80
[32] Belov P A and Simovski C R 2006 *Phys. Rev.* B **73** 045102
[33] Greffet J J and Carminati R 1997 *Prog. Surf. Sci.* **56** 133
[34] Markel V A, Shalaev V M, Zhang P, Huynh W, Tay L, Haslett T L and Moskovits M 1999 *Phys. Rev.* B **59** 10903
[35] Bozhevolnyi S I, Markel V A, Coello V, Kim W and Shalaev V M 1998 *Phys. Rev.* B **58** 11441
[36] Bozhevolnyi S I 1997 *J. Opt. Soc. Am.* B **14** 2254
[37] Carney P S and Schotland J C 2000 *Appl. Phys. Lett.* **77** 2798
[38] Carney P S, Frazin R A, Bozhevolnyi S I, Volkov V S, Boltasseva A and Schotland J C 2004 *Phys. Rev. Lett.* **92** 163903
[39] Chaumet P C, Belkebir K and Sentenac A 2004 *Opt. Lett.* **29** 2740
[40] Belkebir K, Chaumet P C and Sentenac A 2005 *J. Opt. Soc. Am.* A **22** 1889
[41] Bao G and Li P 2007 *Opt. Lett.* **32** 1465
[42] Dorofeenko A V, Lisyansky A A, Merzlikin A M and Vinogradov A P 2006 *Phys. Rev.* B **73** 235126
[43] Kotynski R and Stefaniuk T 2009 *J. Opt. A: Pure Appl. Opt.* **11** 015001