REJOINDER

‘What is negative refraction’: rejoinder

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ARTICLE HISTORY
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ABSTRACT
In this rejoinder, I address some of the points raised by McCall, Kinsler and Favaro in their Response to the preceding Comment.

KEYWORDS
Negative refraction; Poynting vector; Magnetic poles

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1. Definition of negative refraction

In their Reply [1] to Comment [2], McCall, Kinsler and Favaro (MKF) disagree with my statement that the inequality

$$\mathrm{Im}\left(k_t^2\right) < 0 \ , \tag{1}$$

where $\mathbf{k}_t$ is the wave vector of a plane wave refracted from vacuum into a medium across a planar interface, is the general condition for negative refraction. It appears that MKF agree that the condition (1) is applicable to isotropic media and to media with an anisotropic dielectric response, but not to media with magnetic anisotropy. Also, MKF argue that the condition (1) is incompatible with the statement that the Poynting vector in the medium is given (in Gaussian units) by

$$\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{B} \ . \tag{2}$$

MKF’s arguments hinge on two counter-examples which follow equation (15) of the Reply.

In the example (a), MKF consider an isotropic medium with $\epsilon = -1 + 0.1i$ and $\mu = -1$. According to my definition (1), this medium is negatively refracting. MKF do not argue with this. But they write that, if one insists (as I do) that the Poynting vector is given by (2), then “one is obligated to conclude that the net flux [of energy - V.M.] advances towards the interface.” This MKF find to be unphysical. However, I

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have already explained that the example (a) is unphysical. This was one of the main points of my paper [3]. In this paper, I have argued (albeit, on somewhat different grounds) that media with negative refraction, of which the medium (a) is a special case, are unphysical and can not exist in nature. More specifically, I have shown that radiation propagating in the medium (a) would, effectively, push the thermal energy from the medium interior (which would be cooled) to its surface (which would be overheated) and that such energy flux violates the second law of thermodynamics.

Thus, MKF start from the assumption that the medium of example (a) exists, then argue that expression (2) would predict certain unphysical effects in this medium and, based on that, conclude that (2) is incorrect. In reference [3], I have argued that (2) is correct and, if this is so, media of the type (a) can not exist. And, in fact, there are no natural materials even remotely approaching the medium of example (a). There are claims in the literature that artificial structured materials (metamaterials) may have the properties of the medium (a), but I believe that these claims are incorrect because the composites in question are not electromagnetically homogeneous and can not be reasonably characterized by effective medium parameters at the working frequency. It can be mentioned that several recent papers [4–7] investigate the conditions under which metamaterials exhibiting strong magnetic resonances can be assigned effective medium parameters and find that these conditions are rather restrictive.

In the example (b), MKF consider a plane s-polarized wave which is refracted into an anisotropic medium whose optical axes are orthogonal to each other and to the planar interface $z = 0$. The relevant elements of the permittivity and permeability tensors are $\epsilon_\parallel = 1.1 + 0.1i$, $\mu_\parallel = -1$ and $\mu_\perp = 1$, where the symbols "$\parallel$" and "$\perp$" denote the directions parallel and perpendicular to the interface. The dispersion equation for the wave described above is

$$k_{tz}^2 = (\omega/c)^2\epsilon_\parallel\mu_\parallel - k_x^2(\mu_\parallel/\mu_\perp) = -(\omega/c)^2\epsilon_\parallel + k_x^2 \ , \qquad (3)$$

where k_x is the projection of the incident wave vector onto the interface.

According to my definition (1), the medium (b) is negatively-refracting for all values of k_x, but according to the MKF's negative phase velocity (NPV) condition, it is positively refracting for $k_x > 0.04\omega/c$. What is the case in reality? It can be easily seen that for $k_x \lesssim (\omega/c)\sqrt{\mathrm{Re}\epsilon_\parallel}$, the transmitted wave decays exponentially into the medium. According to my definition, negative refraction still occurs, albeit in a formal sense. In any event, arguing whether refraction in this case is positive or negative is rather pointless because, for these values of k_x, there is no noticeable refraction at all. However, for $k_x \gtrsim (\omega/c)\sqrt{\mathrm{Re}\epsilon_\parallel}$, the refraction is definitely negative. Indeed, it can be seen that, in this case, $\mathrm{Im}(k_{tz})\mathrm{Re}(k_{tz}) < 0$. Since $\mathrm{Im}(k_{tz}) > 0$ (exponential decay of any wave transmitted into a passive medium is an incontrovertible experimental fact), we must conclude that $\mathrm{Re}(k_{tz}) < 0$. Therefore, the wave vector of the transmitted wave points towards the interface. This phenomenon is negative refraction according to the classical definition of Mandelstam [8] and Sivukhin [9].

Note that an incident wave which is refracted into the medium (b) from vacuum is evanescent for $k_x \gtrsim (\omega/c)\sqrt{\mathrm{Re}\epsilon_\parallel}$. However, one can also consider the case when the wave is refracted from a transparent medium with the permittivity ϵ_1 such that $\mathrm{Re}\epsilon_1 > \mathrm{Re}\epsilon_\parallel$ and $\mathrm{Im}\epsilon_1 \ll \mathrm{Re}\epsilon_1$. Then there exists a range of k_x in which both the incident and the transmitted wave vectors are approximately real and lie on the same side of the normal to the interface. Yet the NPV criterion would still predict in this case positive refraction, clearly in error.

Regarding the discussion of figure 4 of the Reply, it is not correct that, for sufficiently large values of k_x, the projection of the magnetic field in the medium onto the X-axis vanishes so that the magnetic properties of the medium (given in this example by the tensor element $\mu_\parallel = \mu_{xx} = -1$) become unimportant. The projection turns to zero for just one single value of k_x. Assuming for simplicity that $\epsilon_\parallel = 1 + i\delta$ $(0 < \delta \ll 1)$, this happens exactly at the grazing incidence when $\mathrm{Re}k_{tz} = 0$. In this case, it is not possible to tell whether refraction is positive or negative. This possibility is mentioned in the Comment (see footnote 1). But when k_x is further increased, the X-projection of the magnetic field in the medium becomes again nonzero. As shown above, this leads to the emergence of negative refraction.

It should be clarified that, according to the conclusions of Ref. [3], the medium (b) is also unphysical. Therefore, the arguments given above are hypothetical. I argue that, *if the medium (b) existed*, it would be negatively-refracting by the classical definition, in agreement with the criterion (1).

Also, MKF suggest that the example of an ideal medium with $\epsilon = \mu = -1$, where the imaginary parts of ϵ and μ are identically zero, poses a difficulty for my definition. However, it is well known that Maxwell's equations in an infinite domain have a unique solution only if boundary conditions at infinity are specified. Imposition of the experimentally-verified radiation boundary conditions at infinity (also known as the Sommerfeld boundary conditions) is mathematically equivalent to adding an infinitesimal positive imaginary part to ϵ. Therefore, ϵ has always a small imaginary part. In the case of vacuum, one can take $\epsilon = \epsilon' + i0$, where ϵ' is purely real and $+i0$ is an infinitesimal imaginary part.

2. Form of the Poynting vector

I agree that equalities (16) and (17) of the Reply are identical rearrangements of the same equation. However, the subject of the controversy was physical interpretation of various terms in these equalities. In response to MKF's Reply, it can be pointed out that there are no "effective currents" in the electrodynamics of continuous media - just the macroscopic current of electric charge $\mathbf{J} = \partial\mathbf{P}/\partial t + c\nabla \times \mathbf{M}$. Since the magnetic monopoles do not exist, there can be no macroscopic current of such monopoles. And if such current does not exist, one should not draw any physical conclusions from the assumption that it does. Surely, all experimentally-observable physical phenomena can be explained without ever alluding to nonexistent particles.

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